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CALCULATION OF THE GAUSSIAN BEAM LIGHT SPOT CENTRE DISPLACEMENTS ON MIRROR SURFACES IN MISALIGNED RESONATOR OF LASER GYRO

1 Introduction

Among the main types of optical resonators that are used in constructions of ring laser gyros (RLGs) one can highlight a class of N-corner planar resonators which are formed by spherical mirrors with different curvature radii and by empty space sections. In their arms, such resonators may contain plane-parallel plates which serve, for example, as output Brewster's windows of gas discharge tubes [1-4].

In a perfectly performed RLG resonator, in which the mirrors are adjusted ideally, an axis contour [5–9] coincides with a nominal optical axis and meets the reflecting surfaces of the mirrors in their centres. (The axis contour is a longitudinal axis of symmetry of the Gaussian beam. In any cross section of the beam, the axis contour determines the centre of light spot.)

But in a real RLG resonator, which will be called below as a "misaligned resonator", – the mirrors are slightly shifted (linearly and angularly) from their nominal positions due to various reasons. As a consequence, in such resonator, the axis contour does not coincide with the nominal optical axis, and it meets the reflecting surfaces of the mirrors in other points which are not located in their centres.

2 Formulation of a task

Consider the following task: calculate the Gaussian beam light spot centre displacements on surfaces of the mirrors when RLG resonator is misaligned. This task must be solved in first order in linear and angular misalignments of the mirrors. The results of solving of this task may be used, for example, in theoretical analysis of a behavior of backscattering coefficients in widely known system of RLG dynamic equations [10–12] under condition that device resonator geometry is changing by special lock-in zone control system [2, 3, 12].

An analysis of literature has shown that for RLG resonator of above mentioned type, the named task is not completely solved by this day. There is only one paper [13] in which solution of this task is presented, but the results obtained in it are related only to an empty N-corner RLG resonator of a regular shape.

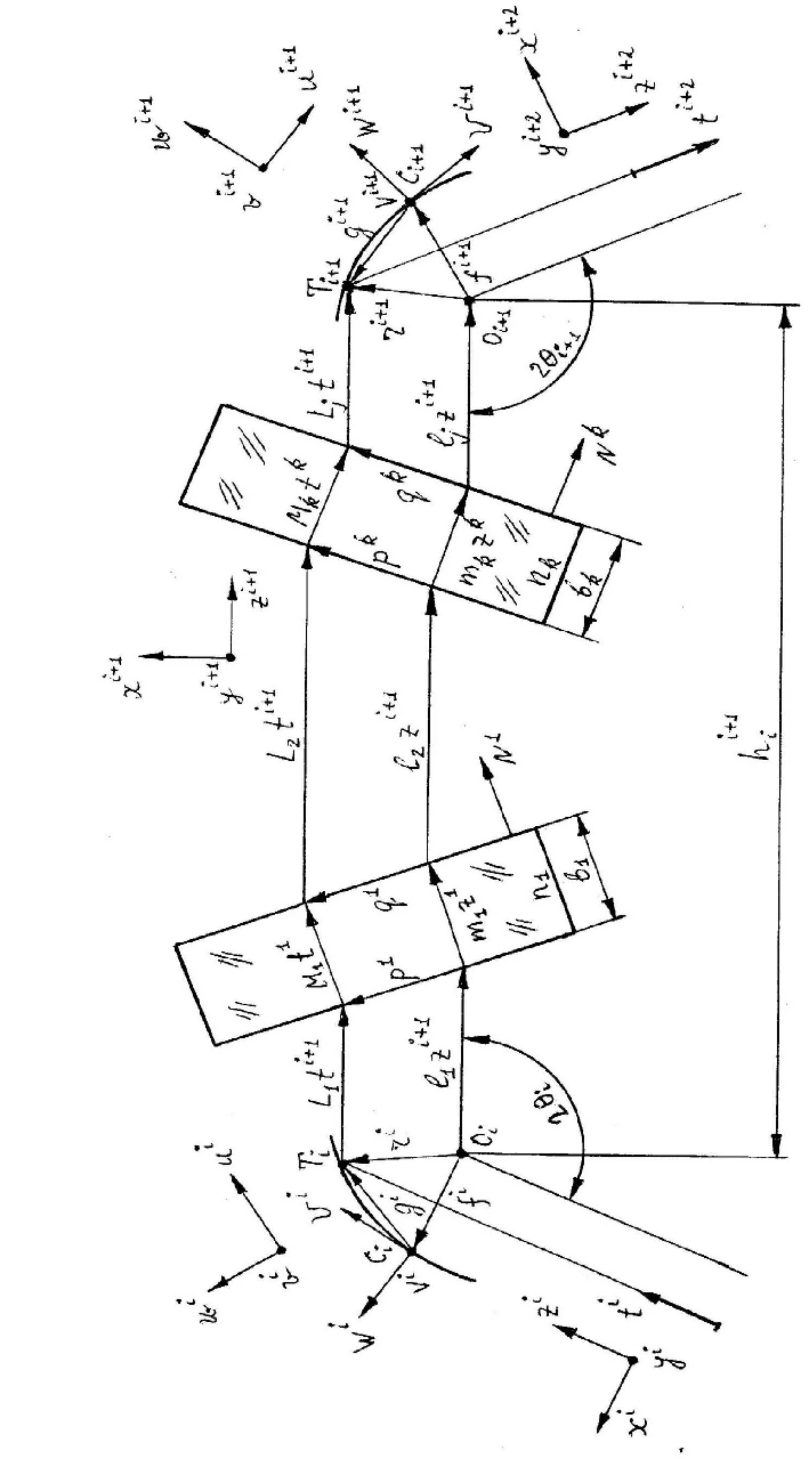

Fig. 1. Optical scheme of RLG resonator arm

3 Description of an optical scheme of RLG resonator arm

Optical scheme of an arm of N-corner RLG resonator is presented in fig. 1. The arm is formed by mirrors M_i, M_{i+1} with curvature radii R_i, R_{i+1} respectively.

The arm contains K ideally performed plane-parallel plates and $J = K + 1$ sections of empty space (the case $K = 2$, $J = 3$ is shown).

In fig. 1, the following notations are used:

x^i, y^i, z^i and x^{i+1}, y^{i+1}, z^{i+1} are the unit vectors of coordinate bases $\{x^i\, y^i\, z^i\}$ and $\{x^{i+1}\, y^{i+1}\, z^{i+1}\}$ introduced for representation of orientation of the axis contour within the empty space sections in the i 's and $(i+1)$'s resonator arms respectively. These vectors are related as

$$\begin{aligned} x^i &= -\, x^{i+1}\cos 2\theta_i - z^{i+1}\sin 2\theta_i , \\ y^i &= y^{i+1}, \\ z^i &= x^{i+1}\sin 2\theta_i - z^{i+1}\cos 2\theta_i , \end{aligned} \qquad (1)$$

where $2\theta_i$ is the angle between the named arms;

w^i, v^i, u^i are the unit vectors of coordinate basis $\{w^i\, v^i\, u^i\}$ introduced for representation of linear and angular displacements of mirror M_i (note that vector w^i is directed along the bisector of angle $2\theta_i$):

$$w^i = x^i \sin\theta_i + z^i \cos\theta_i , \qquad v^i = y^i , \qquad u^i = -\, x^i \cos\theta_i + z^i \sin\theta_i \; ; \quad (2)$$

O_i is the point which determines the nominal position of centre C_i of mirror M_i;

T_i is the point where the axis contour meets the reflecting surface of mirror M_i. It is the point on mirror surface where the Gaussian beam light spot centre is located;

$$f^i = w_i\, w^i + v_i\, v^i + u_i\, u^i \qquad (3)$$

is the vector of small linear displacement of mirror M_i (here w_i and u_i are its normal and tangential displacements in the axis resonator plane, and v_i is its binormal displacement in the sagittal resonator plane);

W^i, V^i, U^i are the unit vectors of coordinate basis $\{W^i\, V^i\, U^i\}$ which is tied with mirror M_i and which has its origin in centre C_i of this mirror (note that vector W^i is normal to the reflecting mirror surface):

$$W^i = w^i + \gamma_i\, v^i - \beta_i\, u^i , \qquad V^i = v^i - \gamma_i\, w^i , \qquad U^i = u^i + \beta_i\, w^i . \qquad (4)$$

Here β_i and γ_i are small angular displacements of mirror M_i in the axis plane around vector v^i and in the sagittal plane around vector u^i;

g^i is the vector which characterizes the displacement on mirror surface of the centre of the Gaussian beam light spot with respect to the centre of this mirror;

$$r^i = x_i x^i + y_i y^i + z_i z^i \tag{5}$$

is the vector which characterizes a deviation of point T_i from point O_i. Here x_i and y_i are the linear transversal coordinates of the axis contour in the input to mirror M_i in the axis plane and sagittal plane respectively; z_i is the linear longitudinal coordinate of the axis contour in the input to this mirror;

$$t^i = \phi_i x^i + \psi_i y^i + z^i \tag{6}$$

is the unit pointing vector ($\left|t^i\right| = 1$) of the axis contour in the i 's resonator arm within the empty space sections. Here ϕ_i and ψ_i are the angular coordinates of the axis contour in the input to mirror M_i in the axis plane and sagittal plane;

h_i^{i+1} is the nominal "overall" length of resonator arm;

l_j is the length of the j 's empty space section ($j = 1, ..., J$);

L_j is the length of the axis contour within the j 's empty space section;

b_k and n_k are respectively the thickness and refraction index of the k's plane-parallel plate ($k = 1, ..., K$);

N^k is the unit vector which is normal to plate surface. This vector is collinear with the unit vector z_{pl}^k of coordinate basis $\{x_{pl}^k \, y_{pl}^k \, z_{pl}^k\}$ tied with the plate. In original plate position, the bases $\{x_{pl}^k \, y_{pl}^k \, z_{pl}^k\}$ and $\{x^{i+1} \, y^{i+1} \, z^{i+1}\}$ coincide. Final orientation of basis $\{x_{pl}^k \, y_{pl}^k \, z_{pl}^k\}$ with respect to basis $\{x^{i+1} \, y^{i+1} \, z^{i+1}\}$ is given by two angles. The first turn – by angle A_k – is realized around the unit vector z^{i+1}:

$$x_{pl}^{k^*} = x^{i+1} \cos \mathrm{A}_k + y^{i+1} \sin \mathrm{A}_k , \quad y_{pl}^{k^*} = - x^{i+1} \sin \mathrm{A}_k + y^{i+1} \cos \mathrm{A}_k ,$$
$$z_{pl}^{k^*} = z^{i+1} . \tag{7}$$

Here the unit vectors $x_{pl}^{k^*}$, $y_{pl}^{k^*}$, $z_{pl}^{k'}$ characterize the intermediate orientation of basis $\{x_{pl}^k\, y_{pl}^k\, z_{pl}^k\}$. The second turn – by angle α_k – is realized around the unit vector $y_{pl}^{k^*}$:

$$x_{pl}^k = (x^{i+1}\cos \mathrm{A}_k + y^{i+1}\sin \mathrm{A}_k)\cos\alpha_k - z^{i+1}\sin\alpha_k,$$

$$y_{pl}^k = -x^{i+1}\sin \mathrm{A}_k + y^{i+1}\cos \mathrm{A}_k, \tag{8}$$

$$N^k = z_{pl}^k = (x^{i+1}\cos \mathrm{A}_k + y^{i+1}\sin \mathrm{A}_k)\sin\alpha_k + z^{i+1}\cos\alpha_k.$$

From expressions (8), the next definitions follow: A_k is the azimuthally angle of the plate, α_k is the angle between the perpendicular to its surface and the nominal axis of resonator. (For example, if two plane-parallel plates depicted in fig.1 are considered as the output Brewster windows of the gas discharge tube, then $b_1 = b_2 = b$, $n_1 = n_2 = n$, $\alpha_1 = -\alpha_2 = \alpha = arctg\, n$. If this tube must provide generation of radiation linear polarized in the axis plane then $A_1 = A_2 = 0$. This is the case presented in fig. 1. But if the tube must provide generation of radiation linear polarized in the sagittal plane then $A_1 = A_2 = \pi/2$);

$$z^k = n_k^{-1}[d_k \sin\alpha_k (x^{i+1}\cos \mathrm{A}_k + y^{i+1}\sin \mathrm{A}_k) + (1 + d_k\cos\alpha_k) z^{i+1}] \tag{9}$$

is the unit pointing vector ($\left| z^k \right| = 1$) of the nominal resonator axis within the plane-parallel plate. Parameter d_k in (9) is calculated by formula

$$d_k = (n_k^2 - \sin^2\alpha_k)^{1/2} - \cos\alpha_k; \tag{10}$$

$$m_k = n_k b_k / (n_k^2 - \sin^2\alpha_k)^{1/2} \tag{11}$$

is the geometrical length of the nominal resonator axis inside the plate;

M_k is the geometrical length of the axis contour inside the plate;

$$t^k = z^k + a_x x^{i+1} + a_y y^{i+1} + a_z z^{i+1} \tag{12}$$

is the unit pointing vector ($\left| t^k \right| = 1$) of the axis contour inside the plate. Here,

$$a_x = n_k^{-1}[(1 - c_k \sin^2\alpha_k \cos^2 \mathrm{A}_k)\phi_{i+1} - (c_k \sin^2\alpha_k \sin \mathrm{A}_k \cos \mathrm{A}_k)\psi_{i+1}],$$

$$a_y = n_k^{-1}[-(c_k \sin^2\alpha_k \sin \mathrm{A}_k \cos \mathrm{A}_k)\phi_{i+1} + (1 - c_k \sin^2\alpha_k \sin^2 \mathrm{A}_k)\psi_{i+1}], \tag{13}$$

$$a_z = n_k^{-1}[-c_k(\phi_{i+1}\cos \mathrm{A}_k + \psi_{i+1}\sin \mathrm{A}_k)\sin\alpha_k \cos\alpha_k],$$

where

$$c_k = 1 - \cos\alpha_k \big/ (n_k^2 - \sin^2\alpha_k)^{1/2}\,; \tag{14}$$

p^k and q^k are the vectors characterizing the Gaussian beam light spot centre displacements on the input and output surfaces of the plane-parallel plate.

4 Original expressions

According to [14, 15], a system of $4N$ linear algebraic equations connecting the linear transversal coordinates x_i, y_i, x_{i+1}, y_{i+1} and the angular coordinates ϕ_i, ψ_i, ϕ_{i+1}, ψ_{i+1} of the axis contour in the inputs to adjacent mirrors M_i and M_{i+1} in above described RLG resonator may be written in the form

$$\begin{cases} x_{i+1} + x_i - l_{i,xx}^{i+1}\phi_{i+1} + l_{i,xy}^{i+1}\psi_{i+1} = 2\,w_i\sin\theta_i, \\ y_{i+1} - y_i + l_{i,yx}^{i+1}\phi_{i+1} - l_{i,yy}^{i+1}\psi_{i+1} = 0, \\ \phi_{i+1} + \phi_i - p_i\,x_i = p_i\,(u_i\cos\theta_i - w_i\sin\theta_i) + 2\beta_i, \\ \psi_{i+1} - \psi_i + q_i\,y_i = q_i\,v_i - 2\gamma_i\cos\theta_i. \end{cases} \tag{15}$$

Here $i = 1,\ldots,N$. If $i = N$ then $i+1 = 1$.

In presented system, $p_i = 2\,R_i^{-1}\sec\theta_i$ and $q_i = 2\,R_i^{-1}\cos\theta_i$ are the optical forces of mirror M_i in the axis plane and sagittal plane respectively. Next, in this system

$$\begin{aligned} l_{i,xx}^{i+1} &= \sum_{j=1}^{J} l_j + \sum_{k=1}^{K}(B_k^{**}\sin^2\mathrm{A}_k + B_k^{*}\cos^2\mathrm{A}_k), \\ l_{i,yy}^{i+1} &= \sum_{j=1}^{J} l_j + \sum_{k=1}^{K}(B_k^{**}\cos^2\mathrm{A}_k + B_k^{*}\sin^2\mathrm{A}_k), \\ l_{i,xy}^{i+1} &= l_{i,yx}^{i+1} = \sum_{k=1}^{K}(B_k^{**} - B_k^{*})\sin\mathrm{A}_k\cos\mathrm{A}_k\,, \end{aligned} \tag{16}$$

where

$$B_k^{*} = n_k\,m_k\cos^2\alpha_k \big/ (n_k^2 - \sin^2\alpha_k), \qquad B_k^{**} = m_k / n_k\,. \tag{17}$$

In these expressions, $\sum_{j=1}^{J} l_j$ is the sum of the empty space section lengths in resonator arm between mirrors M_i and M_{i+1}; B_k^{*} and B_k^{**} are the B–elements of two-dimensional $ABCD$–ray matrix of the k's plane-parallel plate considered in coordinate basis turned by the angle A_k around the unit vector z^{i+1}.

5 Concretization of the task

According to fig. 1, vector g^i characterizes (on surface of mirror M_i) the displacement of the Gaussian beam light spot centre T_i with respect to centre C_i of this mirror. In the basis $\{W^i V^i U^i\}$, which is tied with mirror M_i and which has its origin in point C_i, vector g^i may be presented as

$$g^i = h_i W^i + s_i V^i + t_i U^i . \tag{18}$$

Inside a perfectly performed RLG resonator (for which w_i, v_i, u_i, β_i, $\gamma_i = 0$), the axis contour coincides with the nominal optical axis. It meets the reflecting surface of each mirror M_i in its centre C_i. It is clear that $g^i = 0$ in this case.

But inside a misaligned RLG resonator (for which w_i, v_i, u_i, β_i, $\gamma_i \neq 0$), the axis contour is deformed. Now it does not coincide with the nominal optical axis. It meets the reflecting surface of each mirror M_i in point T_i instead of point C_i. In this case, $g^i \neq 0$.

So, the task is to derive, on the basis of system (15), such a new system of linear algebraic equations which will allow to determine in (18) the t_i and s_i components of vector g^i directly. [As it will be shown below, in linear approximation, $h_i = 0$.]

6 Solving of the task

According to fig.1,

$$g^i = r^i - f^i , \tag{19}$$

or, using (3), (5),

$$g^i = x_i x^i + y_i y^i + z_i z^i - w_i w^i - v_i v^i - u_i u^i . \tag{20}$$

From (2)

$$x^i = w^i \sin\theta_i - u^i \cos\theta_i , \qquad y^i = v^i , \qquad z^i = w^i \cos\theta_i + u^i \sin\theta_i . \tag{21}$$

After substitution (21) into (20)

$$g^i = (x_i \sin\theta_i + z_i \cos\theta_i - w_i) w^i + (y_i - v_i) v^i + (- x_i \cos\theta_i + z_i \sin\theta_i - u_i) u^i . \tag{22}$$

From (4)

$$w^i = W^i - \gamma_i v^i + \beta_i u^i, \qquad v^i = V^i + \gamma_i w^i, \qquad u^i = U^i - \beta_i w^i. \tag{23}$$

As a result of substitution (23) into (22), in first order

$$g^i = (x_i \sin\theta_i + z_i \cos\theta_i - w_i)W^i + (y_i - v_i)V^i + (-x_i \cos\theta_i + z_i \sin\theta_i - u_i)U^i. \tag{24}$$

After comparison (24) and (18)

$$\begin{gathered} h_i = x_i \sin\theta_i + z_i \cos\theta_i - w_i, \qquad s_i = y_i - v_i, \\ t_i = -x_i \cos\theta_i + z_i \sin\theta_i - u_i. \end{gathered} \tag{25}$$

In (25), quantity z_i is the linear longitudinal coordinate of the axis contour in the input to mirror M_i. According to formula (33) in [14],

$$z_i = -x_i tg\theta_i + w_i \sec\theta_i. \tag{26}$$

Then, after substitution (26) into (25), $h_i = 0$ and

$$t_i = -x_i \sec\theta_i + w_i tg\theta_i - u_i, \qquad s_i = y_i - v_i. \tag{27}$$

From (27)

$$x_i = -(t_i + u_i)\cos\theta_i + w_i \sin\theta_i, \qquad y_i = s_i + v_i. \tag{28}$$

Finally, by inserting (28) into (15), we obtain the desired system of $4N$ linear algebraic equations with respect to the unknown variables ϕ_i, ψ_i, t_i, s_i which allow to determine directly quantities t_i and s_i:

$$\begin{cases} t_{i+1}\cos\theta_{i+1} + t_i \cos\theta_i + l^{i+1}_{i,xx}\phi_{i+1} - l^{i+1}_{i,xy}\psi_{i+1} = w_{i+1}\sin\theta_{i+1} - w_i \sin\theta_i \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad - u_{i+1}\cos\theta_{i+1} - u_i \cos\theta_i, \\ s_{i+1} - s_i + l^{i+1}_{i,yx}\phi_{i+1} - l^{i+1}_{i,yy}\psi_{i+1} = -v_{i+1} + v_i, \\ \phi_{i+1} + \phi_i + p_i t_i \cos\theta_i = 2\beta_i, \\ \psi_{i+1} - \psi_i + q_i s_i = -2\gamma_i \cos\theta_i. \end{cases} \tag{29}$$

In this system $i = 1,...,N$. If $i = N$ then $i + 1 = 1$.

Thus, system (29) is the result of solving of the given task.

7 Task solutions for more simple configurations of RLG resonator

7.1 Empty N-corner RLG resonator of an arbitrary shape

Such RLG resonator does not contain the plane-parallel plates in its arms. It is formed by spherical mirrors with different curvature radii and by empty space sections, and may have an arbitrary shape (as, for example, isosceles triangle). For such resonator $l^{i+1}_{i,xy} = l^{i+1}_{i,yx} = 0$ and $l^{i+1}_{i,xx} = l^{i+1}_{i,yy} = h^{i+1}_i = l^{i+1}_i$, where l^{i+1}_i is the nominal length of the arm between mirrors M_i and M_{i+1}. In this case the general system of $4N$ equations (29) converts into two separate systems of $2N$ equations with respect to the unknown variables t_i, ϕ_i and s_i, ψ_i:

$$\begin{cases} t_{i+1}\cos\theta_{i+1} + t_i\cos\theta_i + l_i^{i+1}\phi_{i+1} = w_{i+1}\sin\theta_{i+1} - w_i\sin\theta_i - u_{i+1}\cos\theta_{i+1} - u_i\cos\theta_i, \\ \phi_{i+1} + \phi_i + p_i t_i\cos\theta_i = 2\beta_i, \end{cases} \quad (30)$$

$$\begin{cases} s_{i+1} - s_i - l_i^{i+1}\psi_{i+1} = -v_{i+1} + v_i, \\ \psi_{i+1} - \psi_i + q_i s_i = -2\gamma_i\cos\theta_i. \end{cases} \quad (31)$$

Systems (30) and (31) relate to the axis plane and sagittal plane respectively. In these systems $i = 1,\ldots,N$. If $i = N$ then $i+1 = 1$.

7.2 Empty N-corner RLG resonator of a regular shape

Such RLG resonator has a shape of equilateral N–gon (as a rule, $N = 3,4$) and for it $h_i^{i+1} = l_i^{i+1} = l$, $\theta_i = \theta = \pi/2 - \pi/N$, $p_i = 2R_i^{-1}\sec\theta$, $q_i = 2R_i^{-1}\cos\theta$. In this case, from two systems of $2N$ linear algebraic equations (30) and (31), one can obtain two systems of N equations for direct determination of the unknown variables t_i and s_i

$$\{\ (2-\xi_i)t_i + t_{i+1} + t_{i-1} = (w_{i+1} - w_{i-1})tg\theta - 2u_i - u_{i+1} - u_{i-1} - 2l\beta_i\sec\theta, \quad (32)$$

$$\{\ -(2-\eta_i)s_i + s_{i+1} + s_{i-1} = 2v_i - v_{i+1} - v_{i-1} - 2l\gamma_i\cos\theta, \quad (33)$$

where $\xi_i = p_i l$, $\eta_i = q_i l$. In these systems $i = 1,\ldots,N$. If $i = 1$ then $i-1 = N$. But if $i = N$ then $i+1 = 1$.

8 Comparative analysis of obtained results with those known from literature

As it follows from introduction to this paper, a comparative analysis of the obtained here results with those known from literature may be performed only for an empty N–mirror RLG resonator of a regular shape.

Let us consider the results obtained in paper [13] (see formulas (3), (12), (18), (20) in [13]). Using our notations, these results may be presented in the form

$$\{\ (2-\xi_i)t_i + t_{i+1} + t_{i-1} = -(w_{i+1} - w_{i-1})tg\theta - 2u_i - u_{i+1} - u_{i-1} - 2l\beta_i\sec\theta, \quad (34)$$

$$\{ -(2-\eta_i)s_i + s_{i+1} + s_{i-1} = 2v_i - v_{i+1} - v_{i-1} - 2l\gamma_i \cos\theta . \quad (35)$$

A comparative analysis of systems (33) and (35) – which are related to the sagittal plane – shows their exact identity. While an analysis of systems (32) and (34) – which are related to the axis plane – shows the slight difference between them: terms $(w_{i+1} - w_{i-1})tg\theta$ in the right-hand sides of these systems have the opposite signs.

9 Conclusion

In this paper, the general system of $4N$ linear algebraic equations (29) is derived for calculation of the Gaussian beam light spot centre displacements t_i and s_i on reflecting surfaces in N–corner misaligned RLG resonator which is formed by spherical mirrors with different curvature radii, by empty space sections, and which contains the plane-parallel plates in its arms.

If RLG resonator does not contain the plane-parallel plates in its arms and if it has an arbitrary shape – then quantities t_i and s_i may be found from two systems of $2N$ equations (30) and (31) respectively.

If RLG resonator does not contain the plane-parallel plates in the arms and if it has a regular shape – then quantities t_i and s_i may be determined from two systems of N equations (32) and (33). These systems repeat the known results of paper [13] differing from them only by signs before the normal displacements of the mirrors.

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