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## **COMPARATIVE ANALYSIS OF CHARACTERISTICS OF THE VARIOUS ASSEMBLING SCHEMES DYNAMICALLY TUNED GYROSCOPES**

### **Introduction**

During the present time, dynamically tuned gyroscopes (DRG) find wide applications as sensitive elements in orientation systems and mobile objects management [1]. Meanwhile, the two-frame DRGs which can have the various assembling models of flexible gimbals are especially highly valued among experts.

In particular, a "one ring" model of assembling the two-frame DRG with flexible gimbal is used, for example, in such devices as GVK-16, GVK-17 and GVK-18 (developer Ramenskoye Instrument Design Bureau, Russia) [2], while "two rings" model is used in a small-sized DRG series KIND (the developer "scientific research academician V. I. Kuznetsov institute of applied mechanics", Russia), and also in devices of GD series (the developer OAO "NPK – Kiev G. I. Petrovsky factory of automatics", Ukraine) [3].

### **Problem statement**

We will provide a comparative analysis of accuracy and strength characteristics of the two-frame flexible gimbals DRG, made by "one ring" and "two rings" schemes. At the same time, with the purpose of decrease in bulkiness of received analytical dependences, idealized flexible gimbal, which takes in account only the most important elements of the design, will be considered in research.

### **Constructive schemes of two-frame flexible gimbals DRG**

As objects of our research we consider flexible gimbals two-frame DRG, made by "one ring" (fig. 1) and "two rings" (fig. 2) schemes [2] – [3].

Designs of considered flexible gimbals possess symmetry about an axis of self rotation of DRG, and coordinate axes  $Oy_0$  and  $Oz_0$  are working axes of flexible gimbal.

As shows the analysis, the most essential factors influencing the design of flexible gimbals, are the minimal angular stiffness relative to working axes and linear equistiffness which should be provided in flexible gimbals.

Structurally flexible gimbals DRG consist of two frames, located "in parallel" between a rotor and a power shaft. Thus each frame of gimbal fastens to a rotor and a power shaft by means of two pairs of elastic elements.

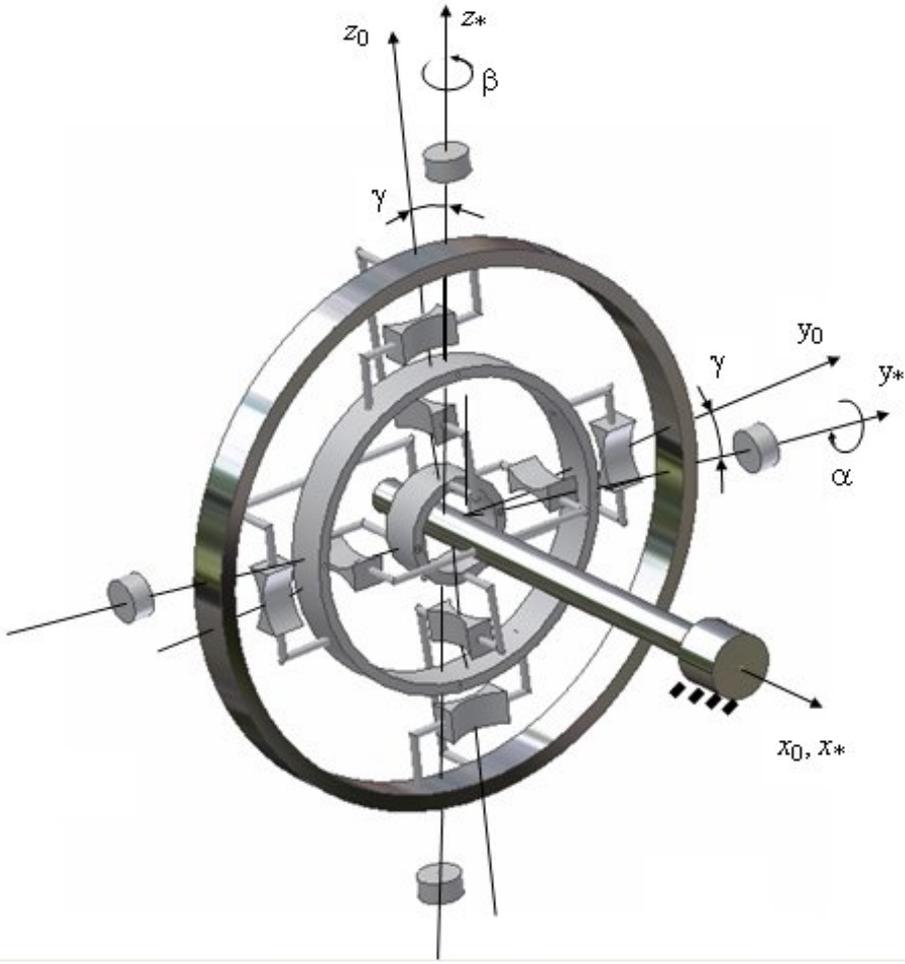


Fig. 1. An arrangement of elastic elements in two-frame flexible gimbal, assembled by "one ring" scheme

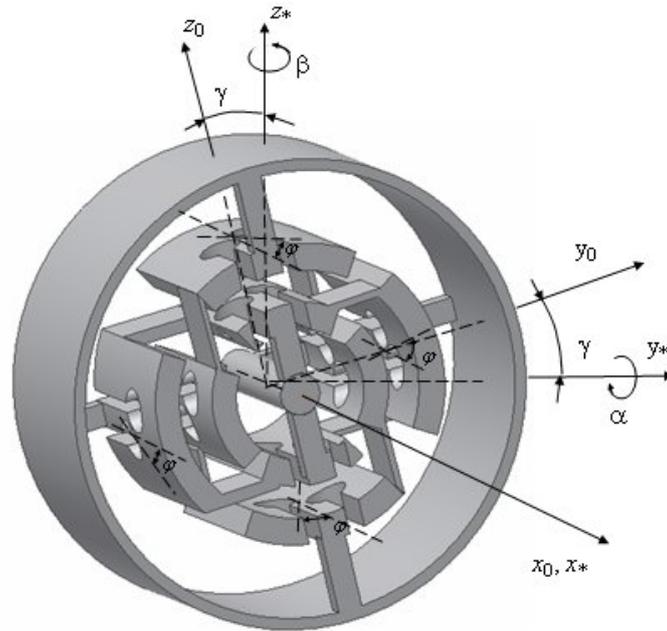


Fig. 2. An arrangement of elastic elements in two-frame flexible gimbal, assembled by "two rings" scheme

The main elements of gimbals' design, substantially defining its basic properties, are elastic elements. In particular, their angular (working) stiffness defines angular stiffness of gimbal relative to its working axes. In considered flexible gimbal elastic elements represent a plate with a variable cross-section, formed by two cylindrical apertures (fig. 3).

Elastic element with sizes, designated according to figure 3:

$b$  – width of an elastic element;

$h$  – thickness of an elastic element in its narrowest part;

$\rho$  – radius of the cylindrical apertures forming an elastic element.

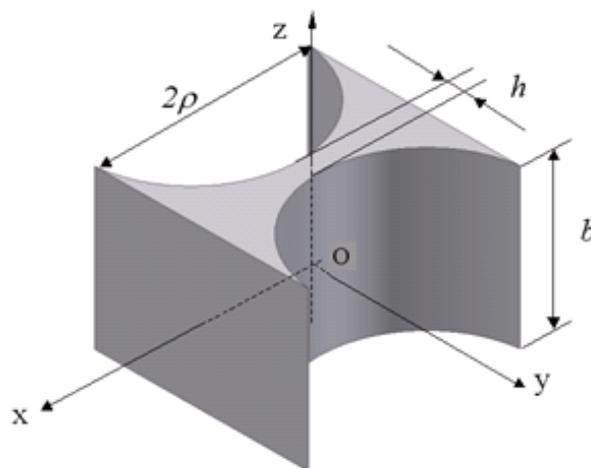


Fig. 3. An elastic element with the variable cross-section, formed by two cylindrical apertures

We also shall notice that for considered elastic elements, the following relations between their geometrical sizes are characteristics are typical:

$$h \ll b, \rho. \quad (1)$$

If we assign, according to figure 3,  $c_i$  ( $i = \overline{1,3}$ ) as coefficients of linear stiffness in directions  $Ox(i=1)$ ,  $Oy(i=2)$  and  $Oz(i=3)$ , and  $c_i$  ( $i = \overline{4,6}$ ) – as coefficients of angular stiffness around axes  $Ox(i=4)$ ,  $Oy(i=5)$  and  $Oz(i=6)$ , we can write down following approached formulas, describing stiffness of elastic elements:

$$\begin{aligned} c_1 &\approx \frac{E_*bh}{\pi L_*}; & c_2 &\approx \frac{2E_*bh^3}{3\pi L_*^3}; & c_3 &\approx \frac{5Gbh}{6\pi L_*}; \\ c_4 &\approx \frac{E_*b^3h^3}{18\pi L_*^3}; & c_5 &\approx \frac{E_*b^3h}{12\pi L_*}; & c_6 &\approx \frac{2E_*bh^3}{9\pi L_*}, \end{aligned} \quad (2)$$

where

$L_* = \sqrt{h\rho}$  – is effective length of an elastic element;

$E_* = \frac{E}{1-\mu^2}$  – is an effective value of the elasticity modulus of a first kind,

considering the lamellar form of an elastic element;

$E$  and  $G$  – are the elasticity modulus of the first and second kind;

$\mu$  – Poisson's ratio of the material.

Working angular stiffness of an elastic element (angular stiffness of an element relative to a working axis of gimbal) is stiffness  $c_6$ , which is the least of angular stiffness's (2). The comparative analysis of the elastic elements' stiffness's, described by the approximated analytical formulas (2), allows, according to expression (1), to receive the following expressions for typical disproportions between elastic element's stiffness in various directions:

$$c_2 \ll c_3 < c_1; \quad c_6 \ll c_4 \ll c_5. \quad (3)$$

From the considered design of flexible gimbal (fig. 1–2), it is easy to see, that angular stiffness relative to working axes  $Oy_0$  and  $Oz_0$  completely specifies the working stiffness of elastic elements. Hence, the relation of the sizes (1) in elastic elements causes fulfillment of one of the basic requirements – the minimality of size of angular stiffness relative to working axes of flexible gimbal.

Now, let's consider the features which bring the condition validity linear equistiffness in designs of two–frame flexible gimbal.

We should note, that invalidity of the given requirement causes error in DRG which cannot be adjusted, and for its elimination gimbals is designed with equistiffness in axial ( $c_{11}^{(n)}$ ) and radial ( $c_{22}^{(n)}$ ) directions:

$$c_{11}^{(n)} = c_{22}^{(n)}. \quad (4)$$

Firstly we will consider the design of idealized flexible gimbal, assembled by "one ring" scheme (fig.1). These flexible gimbals consist of the internal and external frames. The internal frame of the flexible gimbal fastens to a rotor and a power shaft by two pairs of elastic elements working on a bend, with its longitudinal axes be settled down in parallel to the axis of a drive's shaft. On the other hand, two pairs of elastic elements of flexible gimbal external frame, have perpendicular (to an axis of a drive's shaft) arrangement of longitudinal axes.

To estimate between parameters of elastic elements of two constructive details of the flexible gimbal, which are necessary for approached equality correctness (4), we assume, that the elements of internal and external frames' differ only by values of their widths  $b^{(e)}$  and  $b^{(h)}$ , and that for their stiffness' description formulas (2) are correct. Then we can write down the following ratio of widths of elastic elements for external and internal frame [2], required for realization of an equistiffness condition in flexible gimbals (4):

$$\frac{b^{(h)}}{b^{(e)}} \approx 0,5 + \frac{1,2}{1-\mu}. \quad (5)$$

According to the formula (5), the width of elastic elements in external frame should surpass the width of elastic elements in internal frame. In this case we get fulfillment of a linear equistiffness condition in flexible gimbals [2]:

$$c_{11}^{(n1)} \approx c_{22}^{(n1)} \approx c_1^{(e)}, \quad (6)$$

where  $c_{11}^{(n1)}$  and  $c_{22}^{(n1)}$  – are axial and radial stiffness of flexible gimbal, assembled by "one ring" scheme;  $c_1^{(e)}$  – linear stiffness of an elastic element of the internal frame, described by the formula (2).

Stiffness characteristics in gimbals will depend not only on parameters of elastic elements, but also on the scheme of their arrangement. In two-frame DRG with flexible gimbals, assembled by "two ring" scheme, all the elastic elements have identical nominal sizes (fig.2), and linear equistiffness of flexible gimbal is provided by a choice of angular position of elastic elements in flexible gimbal (longitudinal lines of elastic elements with axes, collinear to axis  $Ox_0$  of self rotation of a gyroscope, will form angles  $\phi$ ) [3].

Researches show, that linear equistiffness (4) in idealized flexible gimbals, assembled by a "two ring" scheme, is reached at the following value of an angle  $\phi$  (fig. 2):

$$\phi \approx 35^{\circ}15' . \quad (7)$$

In this case axial ( $c_{11}^{(n2)}$ ) and radial stiffness ( $c_{22}^{(n2)}$ ) of flexible gimbal in approached form are described by the following expression:

$$c_{11}^{(n2)} \approx c_{22}^{(n2)} \approx 6c_2 , \quad (8)$$

where  $c_2$  – is the linear stiffness of an elastic element described by the formula (2).

The analysis of formulas (6), and (8) with taking in account relations (3) shows the essential superiority of linear stiffness in equistiffness flexible gimbals, assembled by a "one ring" scheme, in compare to the similar stiffness of flexible gimbals, assembled by a "two ring" scheme. This circumstance in condition of intensive vibrations and large overloads should cause worse accuracy characteristics of two–frame DRG with "two ring" assembling scheme, in comparison with DRG, possessing flexible gimbals, assembled by a "one ring" scheme.

### **The comparative analysis of errors in DRG with two–frame flexible gimbals assembled by various schemes**

#### **– *The residual stiffness arising during the work of DRG***

Regular operating mode of DRG is close to an operating mode of a free gyroscope [4]. This operating mode is reached due to the satisfaction of the dynamic adjustment condition, which assumes the mutual equilibrium of operating, with respect to working axes of gimbal, moments, arising due to elastic forces and forces of inertia in frames of flexible gimbal.

If we are dealing with the case of violation of dynamic adjustment condition, DRG, similarly to three–level girotachometer, starts to measure a cross component of angular speed of the base. Therefore, the exact satisfiability of the dynamic adjustment condition, assuming zero size of residual stiffness  $\Delta c_0$ , is an important condition of effective work of DRG. During the work of DRG external factors can become the reason of change of angular stiffness in flexible gimbal respectively to its working axes, occurrences of residual stiffness and as a result of infringements of the dynamic adjustment condition lead to inexact work of gyroscope device.

Let's analyze major factors which can lead to occurrence of residual stiffness in two–frame flexible gimbals DRG, applying various assembling schemes.

In DRG with flexible gimbals, assembled by "two ring" scheme, the essential increase in angular stiffness (so–called «jamming») occurs at enough large angles  $\theta$  and  $\psi$  of a turn of a rotor around the working axes of flexible gimbal. Residual stiffness will be described in this case by following expression:

$$\Delta c_0 \approx \frac{3}{2} c_2 r^2 (\psi^2 + \theta^2),$$

where

$c_2$  – is the linear stiffness of an elastic element described by the formula (2);

$r$  – is an average distance from the centers of elastic elements up to the center of flexible gimbal.

On the other hand, for DRG with flexible gimbals, assembled by “one ring” scheme, change in gimbals’ angular stiffness with respect to its working axes occurs during the action of axial force  $P_{x0}$  produced from a rotor’s side, which is caused by a specialty of fastening elastic elements in a design of an internal frame, and leads to their deformation. Residual stiffness will be described in this case by following expression [5]:

$$\Delta c_0 \approx \frac{12}{\pi^2} P_{x0} \sqrt{\rho^{(\theta)} h^{(\theta)}}.$$

**– Errors in DRG in presence of vibration of the base with double frequency**

At the angular vibration of the basis ( $\omega_{z^*} = \omega_{z^*}^{(2\dot{\gamma})} \cos 2\gamma$ ), with the frequency equal to doubled frequency of self rotation, two-frame DRG, with flexible gimbals assembled by “one ring” model, will have the constant component of error  $\langle \dot{\beta}^{(2\dot{\gamma})} \rangle$ , described by the following expression:

$$\langle \dot{\beta}^{(2\dot{\gamma})} \rangle \approx -\frac{1}{4H_0} (a_{x0}^{(\theta)} + a_{x0}^{(H)}) \cdot \dot{\gamma} \cdot \omega_{z^*}^{(2\dot{\gamma})},$$

where

$a_{x0}^{(\theta)}$  and  $a_{x0}^{(H)}$  – are the moments of inertia with respect to axis  $Ox_0$  in an internal and external frames of flexible gimbal;

$H_0$  and  $\dot{\gamma}$  – are self kinetic moment and frequency of self rotation of a gyroscope.

It turns out, that idealized DRG, assembled by “two ring” scheme, will have a constant component of error  $\langle \dot{\alpha}^{(2\dot{\gamma})} \rangle$  during the translational vibration of the base ( $W_{z^*} = W_{z^*}^{(2\dot{\gamma})} \cos 2\gamma$ ), occurring with the frequency equal to doubled frequency of self rotation of DRG [3]. This error in considered case of idealized flexible gimbals will have the following approached view:

$$\langle \dot{\alpha}^{(2\dot{\gamma})} \rangle \approx \frac{c_6}{3\sqrt{2}c_2 r} \cdot \frac{m_0}{H_0} W_{z^*}^{(2\dot{\gamma})}, \quad (1)$$

where

$c_2$  and  $c_6$  – are corresponding stiffness, described by expressions (2), of elastic elements;

$m_0$  and  $r$  – weight of a rotor and average distance from elastic elements up to the center of flexible gimbal.

– ***Quadrature error, caused by technological displacement of elastic elements of gimbal***

Quadrature error of DRG is one of especially undesirable errors, as it cannot be adjusted. The analysis shows, that symmetry in idealized design of flexible gimbal, made by "one ring" model, causes an absence of cross bonds in it, including "quadrature" pendulosity. However it is necessary to notice, that technological factors break symmetry of flexible gimbal and as a consequence, lead to occurrence of various elastic cross bonds.

The researches of technological displacement influence in elastic elements [2], relative to its nominal positions, on elastic cross bonds of gimbal, show that quadrature pendulosity can occur in considered flexible gimbals. Meanwhile, among technological imperfections in gimbals, the most essential influence is caused by small linear  $h_x^{(H)}$  and angular  $\phi_x^{(H)}$  displacement of elastic elements of an external frame, occurring in a direction and relative to their longitudinal lines.

In this case (with technological displacement of one elastic element of an external frame in flexible gimbal) arises quadrature error  $\langle \dot{\alpha}^{(K\theta)} \rangle$  described by the following expression:

$$\langle \dot{\alpha}^{(K\theta)} \rangle \approx -\frac{m_0 W_{z^*}}{12 H_0} \left( \frac{b^{(H)}}{r^{(H)}} \right)^2 \cdot h_x^{(H)} \phi_x^{(H)},$$

where  $r^{(H)}$  – is the distance from the centers of elastic elements of an external frame in flexible gimbal up to its center.

Let's note, that a design of two-frame flexible gimbal, with "two ring" assembling without technological deviations, causes occurrence of quadrature pendulosity in each of frames, identical by size and opposite by a sign. Therefore caused by this factor quadrature error has no constant component. Also, small technological displacements will not result in scalable quadrature errors.

**The comparative analysis of durability of two-frame flexible gimbals, assembled by different models**

It is obvious, that elastic elements of gimbal are critical, from the point of view of durability, elements of a design of dynamically tuned gyroscopes. Meanwhile the most dangerous kind of loading, acting on flexible gimbal, is the inertial force arising in DRG during the accelerated motion of the base.

Our researches have shown that durability of flexible gimbal, assembled by "one ring" scheme, considerably surpasses durability of flexible gimbal, assembled by "two-ring scheme [6]. In particular, we can consider, for example, influence of loading  $P_{x0}$  in an axial direction of two-frame flexible gimbals, having the various assembling schemes.

In flexible gimbal, designed by "one ring" model, the dangerous section with the maximal pressure  $\sigma_{\max}^{(n1)}$  is in the middle of an elastic element of an internal frame. The size of this pressure can be approximated by the following formula:

$$\sigma_{\max}^{(n1)} \approx \frac{P_{x0}}{2b^{(\varepsilon)}h^{(\varepsilon)}}. \quad (9)$$

On the other hand, flexible gimbal, designed by "two ring" model, has dangerous sections with the maximal pressure  $\sigma_{\max}^{(n2)}$  on periphery of elastic elements. The size of pressure in this case approximately can be found by the formula:

$$\sigma_{\max}^{(n2)} \approx \frac{9P_{x0}\sqrt{h\rho}}{32bh^2}. \quad (10)$$

Comparing formulas (9) and (10), it is easy to see that pressures in flexible gimbals, with "two ring" assembling, considerably surpass pressures, arising in flexible gimbals with "one ring" assembling.

## Conclusions

Comparative analysis, which has been done for characteristics of two-frame DRG with flexible gimbals, assembled by "one ring" and "two ring" schemes, allows us to make the following conclusions:

Due to the similarity of elements in flexible gimbals of two-frame DRG, assembled by various schemes, laboriousness and technology of their fabrication are approximately the same.

The geometrical sizes of elastic elements and their arrangement in flexible gimbal are determined, basically, by requirements of minimality of working angular stiffness of elements, and also realization of a linear equistiffness condition in flexible gimbal. Also, the reached size of linear stiffness in flexible gimbal, designed by "one ring" scheme, is essentially above the similar stiffness, attained in flexible gimbal, designed by "two ring" scheme.

During the work of DRG, assembled by "two ring" scheme, it is possible to have a significant increase of residual stiffness (effect of "jamming").

In DRG, assembled by "one ring" scheme, occurrence of residual stiffness is possible due to the change of working angular stiffness of elastic elements of the internal frame, caused by the action of longitudinal force.

For DRG, made by various schemes, it is inherent to have permanent errors caused by vibrations of the base with double frequency, and while in DRG, assembled by "one ring" scheme the permanent error arises due to the angular vibration of the basis, in DRG, assembled by "two ring" scheme, the permanent error occurs at translational vibration.

DRG, with "one ring" assembling is subject to a more significant influence of technological deviations on size of quadrature error, in comparison with DRG, with "two-ring" assembling.

Durability of "one ring" flexible gimbals is significantly higher, than that of "two ring" flexible gimbals.

*Therefore, based on the stated comparative analysis of characteristics of two-frame DRG, we can conclude that for work in conditions of significant overloads it is more preferable to use DRG with "one ring" assembling scheme of flexible gimbal.*

*On the other hand, at the sparing operating conditions usage of DRG with "two ring" assembling scheme of flexible gimbal is more preferable.*

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