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CALCULATION OF TRANSMISSION PROCESSES OF THE AIRCRAFT'S LANDING GEAR

Introduction

The theory of oscillations has taken the considerable interest of scientists because of rapidly increasing machine's capacities and rate of movement of mechanisms stabilisation and controllability of systems. In certain cases, the oscillations may be extremely dangerous for various mechanisms and cause their fault work, rise their wear and noticeably decrease their reliability and cause possibility for breakages and accidents.

I. K. Kosko investigated oscillations in mechanisms in transmission processes [1]. The operetion of any machine may be divided into three periods: start, steady work and deceleration [2]. The transmission processes occur in machines during the periods of their acceleration and deceleration. A transmission process can be defined as the changes of dynamic system oscillation amplitude per a unit of time which occur when the dynamic system changes the mode of its work.

Mechanism's steady motion is a rather long period of time. On the contrary, transmission processes are short periods the last fractions of seconds [1, 2, 3] but stresses at these periods may be much greater than those at steady work. Therefore, it is important to calculate transmission processes and check the ratability of details each time when a new system is developed. Such calculations also help to find the maximum impact on people who work within the machines (miners, pilots and astronauts).

The present paper reports about the possibilities to apply Kosko's theory [1] for aircraft landing dynamic load analysis. The theory developed by this scientist allows us to carry out the mechanism ratability analysis and determine the peak loads on a pilot. The current article aims at investigating transformation processes at aircraft landing.

Problem Statement

The mechanical system design scheme of an aircraft landing gear may be simplified as follows:

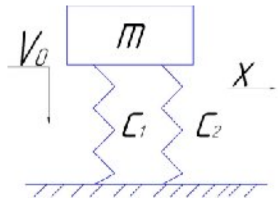

Fig. 1. Scheme to calculate the dynamic factor

The scheme allows us to calculate the dynamic factor η, that is

$$\eta = \frac{P_{dyn}}{P_{st}},$$

where P_{dyn} – a dynamic component

P_{st} – a static component.

This dynamic factor enables to calculate the peaks of maximum stresses in the mechanical system of an airplane landing gear.

Mathematical Model and its Calculation

Let consider the force acting with vertical speed (V_0) on an aircraft landing gear at landing. The issues are represented in fig. 1: x – elongation of the landing gear or displacement of mass m from the point of equilibrium; c_1 and c_2 – the rigidity of landing gear springs.

Elastic force $P_{1\text{-}2}$ is dependent on displacement x and rigidity c. Then

$$P_{1-2} = -cx,$$

When the system starts its work or does deceleration, the oscillations appear in the system. The above-mentioned equality shows that the disturbing forces are not acting; it means that the oscillations in the system are damping. We employ the differential equation to express the displacement of mass m from the point of equilibrium, as follows:

$$m\ddot{x} + b\dot{x} + cx = mg, \tag{1}$$

where b – damping factor, g – free fall acceleration.

This equation is divided by mass m and resulted in the following expression:

$$\ddot{x} + 2h\dot{x} + \omega_0{}^2 x = g, \tag{2}$$

where

$$2h = \frac{b}{m}, \qquad \omega_0{}^2 = \frac{c}{m},$$

where h – damping rate.

We employ operational calculus to integrate differential equation (2) and formulate it as follows:

$$p^2X(p)+2hpX(p)+\omega_0{}^2X(p)=\frac{g}{p}.$$

$$X(p)\left[p^2+2hp+\omega_0{}^2\right]=\frac{g}{p}.$$

$$X(p)=\frac{g}{p(p^2+2hp+\omega_0{}^2)} \qquad (3)$$

We express the right part of equation (3) as a sum. Then

$$\frac{g}{p(p^2+2hp+\omega_0{}^2)}=\frac{A}{p}+\frac{Bp+C}{p^2+2hp+\omega_0{}^2},$$

where *A, B, C* – coefficients to solve the equation. Then

$$g=A(p^2+2hp+\omega_0{}^2)+Bp^2+Cp$$

$$g=Ap^2+2Ahp+\omega_0{}^2A+Bp^2+Cp \qquad (4)$$

The coefficients' values from equality (4) in case of p^2 then in cases of p and p^0 give us system (5):

$$A+B=0,$$

$$2hA+C=0,$$

$$\omega_0{}^2A=g. \qquad (5)$$

From system (5) we formulate the expressions for coefficient *A*, coefficient *B*, and coefficient *C*:

$$A=\frac{g}{\omega_0{}^2},\quad B=-\frac{g}{\omega_0{}^2},\quad C=-\frac{2hg}{\omega_0{}^2}.$$

This allows us to transform equality (3) as follows:

$$X(p)=\frac{g}{p(p^2+2hp+\omega_0{}^2)}=\frac{q}{\omega_0{}^2}\cdot\frac{1}{p}-\frac{q\cdot p}{b^2(p^2+2hp+\omega_0{}^2)}-\frac{2hg}{\omega_0{}^2}\cdot\frac{1}{(p^2+2hp+\omega_0{}^2)}$$

We solve equation (2)

$$x=\frac{q}{\omega_0{}^2}-\frac{q}{\omega_0{}^2}\cdot e^{-ht}(\cos\sqrt{\omega_0{}^2-h^2t}-\frac{h}{\sqrt{\omega^2-h^2}}\sin\sqrt{\omega_0{}^2-h^2t})-$$
$$-\frac{2hg}{\omega_0{}^2}\cdot\frac{1}{\sqrt{\omega_0{}^2-h^2}}\cdot e^{-ht}\cdot\sin\sqrt{\omega_0{}^2-h^2t}.$$

We introduce $\sqrt{\omega_0{}^2-h^2}=\omega$; where ω – angular frequency of damping oscillations. Then, we find solution for x

$$x = \frac{g}{\omega_0^{\,2}}\left[1 - e^{-ht}(\cos\omega t + \frac{h}{\omega}\sin\omega t)\right]. \tag{6}$$

The left part and the right part (equation 6) are both multiplied by *c*, so

$$P_{1,2} = mg\left[1 - e^{-ht}(\cos\omega t + \frac{h}{\omega}\sin\omega t)\right]. \tag{7}$$

This expression represents elastic force $P_{1\text{-}2}$, that is the sum of static component $P_{1\text{-}2st}$ and dynamic component $P_{1\text{-}2dyn}$. The both components are dependent on time:

$$P_{1-2} = P_{1-2st} + P_{1-2dyn}\,.$$

Also here

$$P_{1-2st} = mg,$$

$$P_{1-2dyn} = -mg \cdot e^{-ht}(\cos\omega t + \frac{h}{\omega}\sin\omega t)\,.$$

Calculation Results

When a small value of damping rate $h > \omega_0$, we consider that $h = 0.2\omega$, $\omega = 0.98$. Let the airplane mass be equal to *m*=1 500 *kg*, spring length – *l = 0.5m*, spring diameter – *d=0.07m*. Modulus of elongation for spring is $E = 0.7 \cdot 10^6\ kg/sm^2$. We calculate the spring rigidity $C_1 = C_2 = \frac{EF}{l} = \frac{E\pi d^2}{4l}$. Reduced rigidity of the whole mechanical system equals $C = C_1 + C_2$ (Fig. 1).

Therefore, the changes in the elastic force $P_{1\text{-}2}(t)$ may be represented as follows:

$$P_{1-2} = mg\left[1 - e^{-2{,}68t}(\cos\omega t + 0.2\sin\omega t)\right]. \tag{8}$$

We introduce coefficient $\lambda = \frac{t_1}{T/2}$;

where $T = \frac{2\pi}{\omega}$ – the time period of damping oscillations, t_1 – time of landing gear being loaded with a short-term force. We tabulate the values under λ with 0.5 step:

№	λ	*t*	$P_{1,2}$
1	0	0	0
2	0.5	1.603	1.2*mg*
3	1	3.206	1.533 *mg*

№	λ	t	$P_{1,2}$
4	1.5	4.809	0.8 *mg*
5	2	6.411	0.715 *mg*
6	2.5	8.014	1.2 *mg*
7	3	9.617	1.152 *mg*
8	3.5	11.22	0.8 *mg*
9	4	12.823	0.919 *mg*
10	4.5	14.426	1.2 *mg*
11	5	16.029	1.043 *mg*
12	5.5	17.631	0.8 *mg*
13	6	19.234	0.977 *mg*
14	6.5	20.837	1.2 *mg*
15	7	22.44	1.012 *mg*
16	7.5	24.043	0.8 *mg*
17	8	25.646	0.994 *mg*
18	8.5	27.249	1.2 *mg*
19	9	28.851	1.004 *mg*
20	9.5	30.454	0.8 *mg*
21	10	32.057	0.998 *mg*

The estimation of the dynamic loads at aircraft landing is studied due to the dynamic factor η:

$$\eta = \frac{P_{1-2}}{P_{1-2st}} = \frac{P_{1-2}}{mg},$$

The calculations of η values with MathCAD allow us to plot the graph as follows:

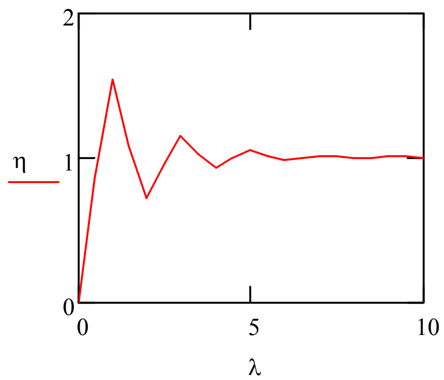

Fig. 2. The calculations of η values with MathCAD

This graph evidences that dynamic factor η has its maximum value at $\lambda = 1.36$. Its peak values from $\lambda=0$ up to $\lambda=5$ are gradually damping. Therefore, from the standpoint of peak loads, the transformation mode is the most dangerous for the mechanical system.

The calculations are carried out with MathCAD. This enables to investigate the behavior of dynamic component of $P_{1\text{-}2dyn}$ force at angular frequency changes ω in the mechanical system. We tabulate the values of $P_{1\text{-}2dyn}$ and ω:

t	$P_{1\text{-}2dyn}$
0	0
0.5	1.2mg
1	1.533 mg
1.5	0.8 mg
2	0.715 mg
2.5	1.2 mg
3	1.152 mg
3.5	0.8 mg
4	0.919 mg
4.5	1.2 mg
5	1.043 mg
5.5	0.8 mg
6	0.977 mg
6.5	1.2 mg
7	1.012 mg
7.5	0.8 mg
8	0.994 mg
8.5	1.2 mg
9	1.004 mg
9.5	0.8 mg
10	0.998 mg

Fig. 3 represents the graph $P_{1\text{-}2dyn}(t)$:

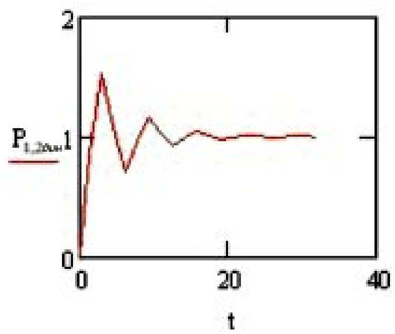

Fig. 3. Represents the graph $P_{1\text{-}2dyn}(t)$

This graph shows that the most loaded periods are transmission processes at $0 \le t \le 20s$. At this time period the dynamic component of elastic forces has great peak loads which start damping at $t \ge 20s$.

Conclusions

1. The methodology to calculate oscillations of transmission processes in aircraft landing gear at landing is developed.
2. Dynamic factor η and λ are determined.
3. Dynamic factor η has the maximum value at $\lambda = 1.36$. Its peak values from $\lambda = 0$ to $\lambda = 5$ are gradually damping. Therefore, from the standpoint of peak loads, the transformation mode is the most dangerous.
4. The investigations determine the fact that the most loaded periods are transmission processes at $0 \le t \le 20s$. At this time period the dynamic component of elastic forces has great peak loads which start damping at $t \ge 20s$.
5. Dynamic addition of $P_{1\text{-}2dyn}$ force has the nature of oscillations accompanied by damping and shows the dependence on angular frequency of oscillations in the mechanic system.

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