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ROBUST ATTITUDE DETERMINATION AND CONTROL SYSTEM OF MICROSATELLITE

Introduction

Improvement of microsatellite control motion is carried out after a few basic directions. The first direction is associated with the use of modern methods of information evaluation concerning microsatellite attitude. For the solution of this problem the Kalman filter is used, as a rule [1]. The second direction includes development and perfection of methods of control moments forming. Using of reaction-wheel control which is more effective than magnetic control, is limited to the necessity of the periodic unloading of reaction-wheels, that complicates a control algorithm substantially. Finally, the third direction consists in drawing on accomplishments of modern theory of automatic control, in particular, powerful methods of robust control [2].

Problem statement

The complex problem of quality increasing of the satellite spatial orientation control is examined by the use of modern methods of information treatment, algorithms of control moments forming and using of robust control methods.

Principle of the control system construction

Let us describe satellite motion with the followings equations:

$$J\dot{\omega}_{BI}^{B} + \omega_{BI}^{B} \times \left(J\omega_{BI}^{B}\right) = \tau^{B},$$
$$\dot{\boldsymbol{q}} = \frac{1}{2}\tilde{\omega}_{BO}^{B} \circ \boldsymbol{q}, \tag{1}$$

where $J = diag(J_x, J_y, J_z)$ is a inertia tensor of satellite; ω_{BI}^{B} is a vector of absolute angular rate of satellite resolved in the body frame; ω_{BO}^{B} is a vector of angular rate of satellite in relation to the orbital frame, resolved in the body frame; $\boldsymbol{q} = \left(\eta, \varepsilon^{T}\right)^{T}$ is quaternion of orientations; τ^{B} is a total mechanical moment, forced on the satellite.

The moments are created with magnetic coils and reaction-wheels. For the exception of the unloading mode of reaction-wheels an integral action is entered in the control law, thus it is given only on coils. For explanation of expedience of such control will write down the simplified model of satellite motion round one axis:

$$J_x\ddot{\varphi}+k_1\dot{\varphi}+k_2\varphi+k_3\int\varphi\,dt=\tau_x, \qquad (2)$$

where k_1, k_2 are coefficients in expression for a moment, which is formed by the reaction-wheel; k_3 it is a coefficient in expression for a moment, which is formed by the coil.

We will write down equation of motion of reaction-wheel as follows:

$$\dot{\mathbf{h}}=k_1\dot{\varphi}+k_2\varphi, \qquad (3)$$

where $\dot{\mathbf{h}}$ is a momentum of reaction-wheel.

From the equations (2), (3) we find expression for a steady value of momentum:

$$\mathbf{h}_{ycm}=\frac{k_2}{k_3}\tau_x. \qquad (4)$$

It is seen that as a result of integral introduction a steady momentum will be limited even at presence of steady moment which is forced on satellite. It is meant that the mode of reaction-wheels unloading is not needed, that substantially simplify a control algorithm.

At the accessible estimation of complete state vector $\left(\hat{\omega}_{BO}^{B\ T},\hat{\boldsymbol{q}}^T\right)^{\mathrm{T}}$ of satellite it is suggested to choose a control moment as:

$$\tau_{cntr}^B=\tau_{RW}^B+\tau_{coils}^B, \qquad (5)$$

where

$\tau_{RW}^B=-K_\omega\hat{\omega}_{BO}^B-K_\varepsilon\hat{\varepsilon}$, $\quad \tau_{coils}^B=\left(L_\omega\hat{\omega}_{BO}^B\times\mathbf{B}^B+L_\varepsilon\hat{\varepsilon}\times\mathbf{B}^B+L_I\int\hat{\varepsilon}dt\times\mathbf{B}^B\right)\times\mathbf{B}^B$ – moments which are created with reaction-wheels and electromagnetic coils accordingly; $\mathbf{B}^B$ is a vector of induction of the Earth magnetic field; $L_\omega, L_\varepsilon, L_I$ are coefficients of energy-based controller; diagonal matrices K_ω and K_ε in obedience to the desired descriptions of step response.

It is suggested to find coefficients energy-based controller on the basis of the mathematical programming methods, that to find them as a solution of the following optimization problem:

$$\min_{\boldsymbol{x}} F(\boldsymbol{x}),$$

$$\text{under restriction}:$$

$$G_i(\boldsymbol{x})\le 0,\ i=1,\dots,m, \qquad (6)$$

where $F(\boldsymbol{x})$ is an objective function, $G_i(\boldsymbol{x})$ are limitations on equations (1) and requirements to the transitional process.

In default of angular rate sensor, it will be to decide the problem of satellite full state vector estimation on the ground of signals only position sensors – magnetometer and Sun sensor with the use of algorithm of Extended Kalman filter or algorithm of set-valued estimation.

Set-valued algorithm of satellite state vector estimation

Essence of set-valued satellite state vector estimation consists in finding of its estimation as to the center of the ellipsoid, which the true value of state vector belongs to assuredly. In it the key difference of this approach consists classic Kalman filtration in approach, where search point estimate of state vector.

For the basis of the construction of procedure of set-valued estimation of state vector an algorithm, described in [3], is taken, where for the estimation of the state of the nonlinear systems higher order terms in the expansion of nonlinear functions in the Taylor series of it is plugged in a guaranteed set which disturbance belongs to.

For the receipt of algorithm of set-valued estimation, we will write down the model of errors of satellite in state space of consisting of kind [4]:

$$\delta\dot{x}(t) = A(t)\delta x(t), \tag{7}$$

where $\delta\boldsymbol{x}^T(t) = \left(\delta\boldsymbol{\omega}^T, \delta\boldsymbol{\varepsilon}^T\right)$ is a vector of errors; $\hat{\omega} \equiv \hat{\omega}_{BO}^B$;

$$A(t) = \begin{bmatrix} -[\hat{\omega}\times] & 0.5I_{3\times3} \\ 2J^{-1}\left(3\omega_0^2 F_g + \left[\boldsymbol{\mu}^B \times\right]\left[\mathbf{B}^B \times\right]\right) & J^{-1}F_e \end{bmatrix};$$

$$\boldsymbol{\mu}^B = L_\omega \hat{\omega}_{BO}^B \times \mathbf{B}^B + L_\varepsilon \hat{\boldsymbol{\varepsilon}} \times \mathbf{B}^B + L_I \int \hat{\boldsymbol{\varepsilon}} dt \times \mathbf{B}^B,$$

$$F_g = \begin{bmatrix} \left(c_{23}^2 - c_{33}^2\right)\left(J_y - J_z\right) & -c_{13}c_{23}\left(J_y - J_z\right) & c_{23}c_{13}\left(J_y - J_z\right) \\ c_{13}c_{23}\left(J_z - J_x\right) & \left(c_{33}^2 - c_{13}^2\right)\left(J_z - J_x\right) & -c_{33}c_{23}\left(J_z - J_x\right) \\ -c_{13}c_{33}\left(J_x - J_y\right) & -c_{23}c_{33}\left(J_x - J_y\right) & \left(c_{13}^2 - c_{23}^2\right)\left(J_x - J_y\right) \end{bmatrix};$$

$$F_e = \begin{bmatrix} 0 & \hat{\omega}_z\left(J_y - J_z\right) & \hat{\omega}_y\left(J_y - J_z\right) \\ \hat{\omega}_z\left(J_z - J_x\right) & 0 & \hat{\omega}_x\left(J_z - J_x\right) \\ \hat{\omega}_y\left(J_x - J_y\right) & \hat{\omega}_x\left(J_x - J_y\right) & 0 \end{bmatrix},$$

Algorithm of set-valued estimation of satellite state vector with the multiplicative update of estimation of quaternion orientation is presented on fig. 1.

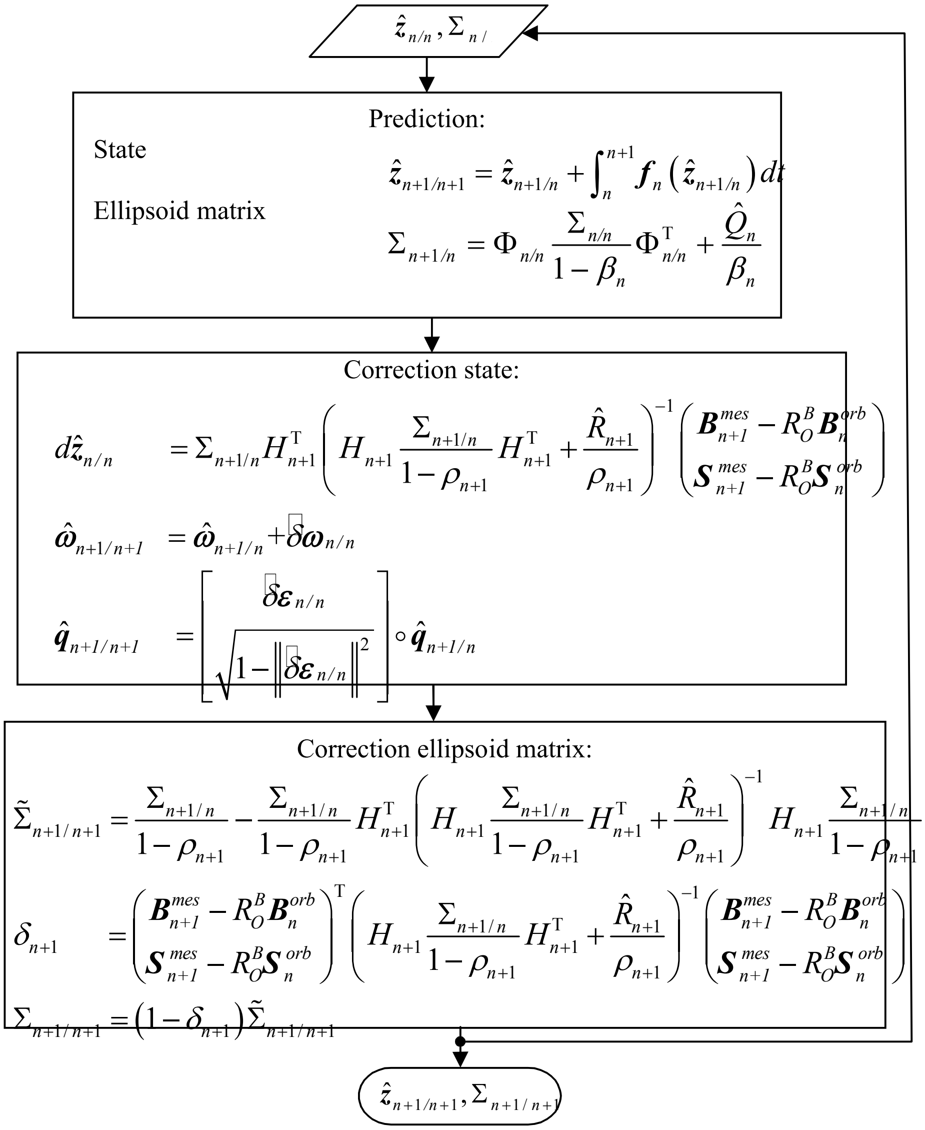

Fig. 1. Multiplicative set-valued filter implementation

Equations of measurement for the system (3) will be written down in a form:

$$z(t) = H\delta x(t) = 2\left[\begin{array}{c|c} \left[\bar{\mathbf{B}}^B(t)\times\right] & 0_{3\times3} \\ \hline \left[\bar{\mathbf{S}}^B(t)\times\right] & 0_{3\times3} \end{array}\right],$$

where $\mathbf{B}^O$ and $\mathbf{S}^O$ – accordingly vector of induction of the Earth magnetic field and Sun-vector, resolved in the orbital coordinate system.

It is noted on fig. 1: $\boldsymbol{f}_n(\hat{\boldsymbol{z}}_{n+1/n}) = \begin{pmatrix} \boldsymbol{J}^{-1}\left(\hat{\tau}_g^B + \tau_{cntr}^B - \hat{\omega}_{BI}^B \times \left(J\hat{\omega}_{BI}^B\right)\right) \\ 0.5\tilde{\omega}_{BO}^B \circ \boldsymbol{q} \end{pmatrix}.$

The initial conditions of algorithm, given in fig.1, are written down in a form: $d\hat{z}_{0/0} \in \Omega\left(0, \Sigma_{0/0}\right), w_{0/0} \in \Omega\left(0, \hat{Q}_{0/0}\right), v_{0/0} \in \Omega\left(0, \hat{R}_{0/0}\right)$.

Numerical design of satellite motion

The numerical analysis of satellite motion is executed for a energy-based controller, expected in obedience to an algorithm (6), at availability of full state vector. The problem was solved with methods of numerical optimization with the use of expansion of Global Optimization Toolbox environments of Matlab.

It was considered that satellite with inertia $J = diag(1{,}44\ 1{,}48\ 0{,}76)$ [kg·m^2] moved on a circle orbit on height of a 650 km with the orbit angle inclination of 98^0. As the model of the Earth magnetic field the model of WMM2005 is accepted [5]. It was considered that satellite in forced permanent revolting moment $\boldsymbol{\tau}_d^B = (1\ \ 2\ \ -2)^T \cdot 10^{-7}$ [N·m].

On fig. 2 the change of Euler angles are built at the suppositions done higher and at the initial conditions of kind $(\varphi, \theta, \psi) = (-20^0, 10^0, 20^0)$. The dotted curves on this graphic corresponds the design of control system with a energy-based controller, the coefficients of which are found by trial-and-error procedure ($L_w = 1{,}5 \cdot 10^{-6}$; $L_s = 1400$), and continuous – to the design with a controller, the coefficients of which are found as a decision of optimization task ($L_w = 1{,}804 \cdot 10^{-6}$; $L_s = 960{,}1$). The dotted curve on fig. 2 according to the estimation of angles orientation received based on set-valued algorithm.

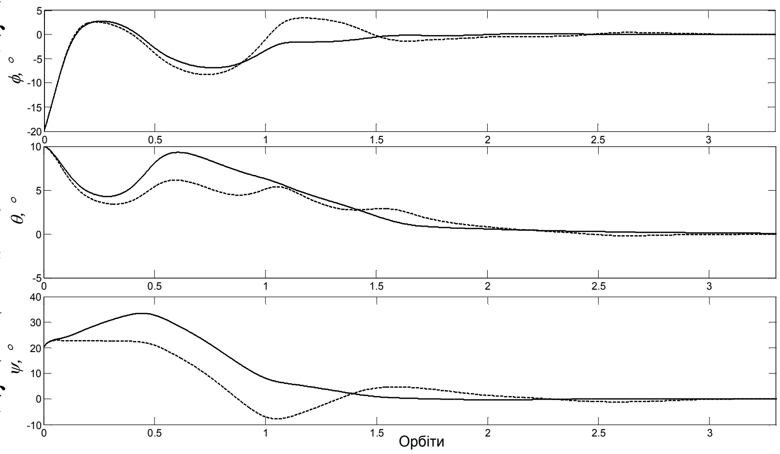

Fig. 2. Behavior Euler angles for energy based controller

From fig. 2 we see, that the energy-based controller, obtained from the solution of optimization problem (6), allows to decrease considerably the time of transitional process comparatively with a regulator, obtained by other method. In addition, such approach allows flexible form performance of control system through the proper choice of limitations $G_i(x) \le 0$ in (6).

Conclusions

Design of the attitude determination and control satellite system with the use of reaction-wheels and electromagnetic coils allows to arrive at accuracy of angular orientation at the level of ten of angular minutes, and application of energy-based controller with an integral action allows to avoid the necessity of procedures implementation of reaction-wheels unloading. Thus, estimation of satellite state vector with multiplicative update of quaternion orientation estimation allows set-valued filter to provide high accuracy of estimation at uncertain environment of operating on a satellite.

The applied method of finding of the control system regulator coefficients allowed substantially to simplify and accelerate the process of its development, providing the high performance of control.

The effort in subsequent researches is expedient to point the set-valued appraiser of the augmented satellite state vector at development, including disturbance moment which is forced on satellite.

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