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THE CALCULATION OF SANDWICH STRUCTURES UNDER ACTION OF STATIC AND DYNAMIC LOADS

Introduction

Sandwich plates and shells are common in the aircraft industry. The problem of designing these structures under static and dynamic loads is very crucial. The solution of this problem was extensively studied by different authors.

The numerical methods are more effective than analytic in solving many practical problems. This paper presents a method for solution of static and dynamic problems which based on variational–grid approach. Distinctive features of it lies in choosing the approximating functions of deflection which are required to reduce the number of variables in comparing with other approaches that using in the calculation of the stress–strain state of sandwich structures [1, 2]. The method–wise descent was used for the minimization of functionals, which were received after the solution of static and dynamic problems.

The calculation of the stress–strain state of sandwich structures based on the hypothesis of a broken line [3], according to which the strain state of thin supporting layers satisfies the Kirchhoff–Loveand hypothesis. The distribution of tangential displacements for the filling is taken along.

Problem statement

Lets consider a three–layer plate whose middle surface occupies a region $\Omega \subset R^2$. The problem of bending in the variational formulation which based on the principle of potential energy can be formulated as a problem of minimizing the functional: to find a displacement *u*, which satisfies the conditions

$$u \in V; \quad Э(u) = inf\, Э(v); \quad v \in V, \tag{1}$$

where V – is the space of admissible displacements, the functional E(v) – is the potential energy.

It is necessary to determine the natural frequencies and corresponding vibration modes in calculating of the stress–strain state of structural elements which are exposed to vibration. The finding of the main natural frequency can be reduced to the minimization problem, where the functional is determined by the ratio of the Rayleigh – Ritz

$$\omega^2 = \min \frac{\Pi(v)}{T(v)}, \quad v \in V. \tag{2}$$

Here is $\Pi(v)$ – is the peak value of strain energy; T(v) – is the quantity which is proportional to the amplitude value of kinetic energy.

Using the hypothesis of a broken line, the kinematic relations for the tangential displacements of a three–layer structure can be written as:

In the upper bearing layer

$$\frac{1}{2}h_3 \le z \le \frac{1}{2}h_3 + h;$$

$$u = u_1 - \left(z - \frac{1}{2}(h_3 + h)\right)w_x; \tag{3}$$

$$v = v_1 - (z - \frac{1}{2}(h_3 + h))w_y.$$

In the filling

$$-\frac{1}{2}h_3 \le z \le \frac{1}{2}h_3;$$

$$u = \frac{u_1 + u_2}{2} + \frac{z}{h_3}(u_1 - u_2 + hw_x);$$

$$v = \frac{v_1 + v_2}{2} + \frac{z}{h_3}(v_1 - v_2 + hw_y).$$

In the lower bearing layer

$$-\frac{1}{2}h_3 - h \le z \le -\frac{1}{2}h_3;$$

$$u = u_2 - (z + \frac{1}{2}(h_3 + h))w_x;$$

$$v = v_2 - (z + \frac{1}{2}(h_3 + h))w_y.$$

Where u_i, v_i – are the median surfaces displacements of the bearing layers along the *x* and *y* axes, respectively; h_3, h – are the thickness of the filler and the base layer; w = *f(x, y)* –is the deflection; the subscripts *x* and *y* are indicate on differentiation with respect to that variable.

According to Hooke's law, stresses in the layers are

$$\sigma_x = \frac{E}{1 - \nu^2}(\varepsilon_x + \nu\varepsilon_y);$$

$$\sigma_y = \frac{E}{1 - \nu^2}(\varepsilon_y + \nu\varepsilon_x);$$

$$\tau_{xy} = C\gamma_{xy}. \qquad (4)$$

In the soft filler

$$\tau_{xz} = G_3\gamma_{xz}, \qquad \tau_{yz} = G_3\gamma_{yz}, \qquad (5)$$

where E – is the modulus of elasticity of the first kind;

G_3 – is the shear modulus;

ν – is the Poisson's ratio.

The relationship between the deformation and displacement is determined by Cauchy relation [4].

Solution of problem

The stacked triangular element was used in constructing the variational - grid schemes for Sandwich structures, which, in contrast to previously developed models [1, 3, 5] apply practically various approximation of displacements for different layers. We used an incomplete cubic polynomial to approximate the deflection of thin supporting words within each triangle:

$$w_h = w_iL_i + w_jL_j + w_kL_k + \alpha_1L_i^2L_j + \alpha_2L_i^2L_k + \alpha_3L_j^2L_i + \alpha_4L_j^2L_k + \alpha_5L_k^2L_i \qquad (6)$$

$+ a_6 L_k^2 L_j + 2a_7 L_i L_j \;_k,$

where are applicable L–coordinate [5] and

$$a_1 = w_i - w_j - b_k\varphi_i - c_k\psi_i,$$

$$a_2 = w_i - w_k + b_j\varphi_i + c_j\psi_i,$$

$$a_3 = w_j - w_i + b_k\varphi_j + c_i\psi_j,$$

$$a_4 = w_j - w_k - b_i\varphi_j - c_i\psi_j,$$

$$a_5 = w_k - w_i - b_j\varphi_k - c_j\psi_k,$$

$$a_6 = w_k - w_i + b_i\varphi_k + c_i\psi,$$

$$a_{i7} = \frac{1}{4}\sum_{s=1}^{6} a_n . \quad (7)$$

Here are $\varphi_i = (\frac{\partial Wi}{\partial y})_j$; $\psi_{ij} = (\frac{\partial Wi}{\partial x})_j$ – the angles of rotation of the plate in the i - node is respectively around the axes of x and y.

$$b_i = y_j - y_k; \quad c_i = x_k - x_j \quad (8)$$

The remaining expressions in (8) are obtained by cyclic permutation of indices.

For filler using a linear approximation of deflection:

$$w_{h3} = w_i L_i + w_j L_j + w_k L_k . \quad (9)$$

To minimize the functional is proposed to use the method of wise descent.

k +1 approximation is constructed in the form

$$\vec{v}^{k+1} = \vec{v}^k + \beta^{k+1}\lambda_i^{k+1}\,\vec{e}_i , \quad (10)$$

where $\vec{v}$ – is the sought displacement vector; $\vec{e}_i$ – is the unit vector in the direction of the components $\vec{v}_i^k$; λ_i^{k+1} – is the step; β – is the relaxation parameter: $\beta \in [1, 2]$.

The criterion to stop the iterative process accepts

$$\|\vec{\lambda}_i^{k+1}\| \le \varepsilon \|\vec{v}_i^{k}\|,$$

where ε – is a some preassigned small constant. The problems of bending and oscillations of sandwich panels with square, rectangular and keystone shapes, which rigidly clamped along the contour were solved as a test.

Conclusions

We note the following positive aspects of the proposed approach. In this three–layer triangular elements we using different approximating functions trough bearing layers and filler, that can significantly simplify the algorithm and reduce the number of arithmetic operations in the numerical implementation, as compared with the elements where the approximation of the deflection are taken identical for all layers [1, 2].Wise descent method is convergent iterative algorithm and rounding errors have little influence on the accuracy of the final result.

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