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Sergei Trubachev, Olga Alekseichuk

CALCULATION OF LAYERED SHELLS WITH ACCOUNT OF THE NONLINEAR DISTRIBUTION OF DISPLACEMENTS

Introduction

Layered shells, which were made of high-strength composite materials with different stacking layers, are widely used in aviation technology as elements of the lifting surfaces of aircraft, as well as in many other industries. Thus, the improving of the calculating methods in heterogeneous layered structures is an actual task. Along with the methods of nondestructive testing, flaw detection of structures using hardware techniques, the numerical methods, which allow predicting the possible collapse of the structure are widely used. There is a considerable number of publications devoted to the calculation of layered structures [1-4], but previously discussed studies did not take into account the possibility of studying the work of each layer separately, which is important when different boundary conditions are tasked on each layer separately.

Problem statement

Lets consider an element of a layered shell, which consists of thin hard layers and soft bearing filler, which is located between them. The displacement of points of bearing layers presented in the form :

$$u^{(n)}(\alpha_1,\alpha_2,z) = u_0^{(n)}(\alpha_1,\alpha_2) + z^{(n)}\theta_1^{(n)}(\alpha_1,\alpha_2)$$

$$v^{(n)}(\alpha_1,\alpha_2,z) = v_0^{(n)}(\alpha_1,\alpha_2) + z^{(n)}\theta_2^{(n)}(\alpha_1,\alpha_2)$$

$$w^{(n)}(\alpha_1,\alpha_2,z) = w_0^{(n)}(\alpha_1,\alpha_2), \quad n = 1,2 \tag{1}$$

Where $\theta_1^{(n)}$ and $\theta_2^{(n)}$, $n = 1,2$ are the rotation angles of the normal in load-bearing layers on the planes:

$$\theta_1^{(n)}(\alpha_1,\alpha_2) = -\frac{1}{A_1}\frac{\partial w_0^{(n)}}{\partial \alpha_1} + \frac{u_0^{(n)}}{R_1},$$

$$\theta_2^{(n)}(\alpha_1,\alpha_2) = -\frac{1}{A_2}\frac{\partial w_0^{(n)}}{\partial \alpha_2} + \frac{v_0^{(n)}}{R_2}, \tag{2}$$

where $u_0^{(n)}$, $v_0^{(n)}$, $w_0^{(n)}$, $n = 1,2$ – are the displacement components of points of the middle surface of the bearing layers in the direction of the coordinate axes in the adopted system of coordinates. R_1 , R_2 –are the respective radii of curvature. We accepted the nonlinear distribution of displacements for the filler.

$$u^{(n)}(\alpha_1,\alpha_2,z)=u_0^{(n)}(\alpha_1,\alpha_2)+\sum_{m=1}^{3}u_i(\alpha_1,\alpha_2)z^m$$

$$v^{(n)}(\alpha_1,\alpha_2,z)=v_0^{(n)}(\alpha_1,\alpha_2)+\sum_{m=1}^{3}v_i(\alpha_1,\alpha_2)z^m$$

$$w^{(n)}(\alpha_1,\alpha_2,z)=w_0^{(n)}(\alpha_1,\alpha_2)+\sum_{m=1}^{3}w_i(\alpha_1,\alpha_2)z^m\ . \qquad (3)$$

The stresses in the bearing layers are represented in the form [1]

$$\{\sigma^{(n)}\}=[G^{(n)}]\{\varepsilon^{(n)}\},\quad n=1,2\ , \qquad (4)$$

where $\{\sigma^{(n)}\}=\{\sigma_1^{(n)},\sigma_2^{(n)},\tau_{12}^{(n)}\}$ -are the components of the stress tensor;

$\{\varepsilon^{(n)}\}=\{\varepsilon_1^{(n)},\varepsilon_2^{(n)},\gamma_{12}^{(n)}\}$ - are the components of strain tensor

$[G^{(n)}]=\begin{pmatrix} g_{11}^{(n)} & g_{12}^{(n)} & g_{13}^{(n)} \\ \dots & g_{22}^{(n)} & g_{23}^{(n)} \\ \dots & \dots & g_{33}^{(n)} \end{pmatrix}$ - is the matrix of elastic coefficients.

We have for an isotropic material bearing layers

$$g_{11}^{(n)}=g_{22}^{(n)}=\frac{E_n}{1-v_2^{(n)}},$$

$$g_{12}^{(n)}=\frac{E_n v_n}{1-v_n^2},\quad g_{33}^{(n)}=G^{(n)},$$

$$g_{13}^{(n)}=g_{23}^{(n)}=0 \qquad (5)$$

For the filler as components of the vectors $\{\sigma^{(3)}\}$ and $\{\varepsilon^{(3)}\}$ that appear

$$\{\sigma^{(3)}\}=\{\tau_{13}^{(3)},\tau_{23}^{(3)},\sigma_3^{(3)}\}\ ,$$

$$\{\varepsilon^{(3)}\}=\{\gamma_{13}^{(3)},\gamma_{23}^{(3)},\varepsilon_3^{(3)}\}. \qquad (6)$$

According to Hooke's law:

$$\{\sigma^{(3)}\}=[G^{(3)}]\{\varepsilon^{(3)}\}. \qquad (7)$$

In the case of coincidence of the axes of orthotropy with the coordinate lines the matrix $[G^{(3)}]$ has a diagonal structure

$$[G^{(3)}]=[g_{55}^{(3)},g_{44}^{(3)},g_{33}^{(3)}]\ , \qquad (8)$$

where for a uniform thickness of the material we have

$$g_{55}^{(3)}=G_{13}^{(3)};\qquad g_{44}^{(3)}=G_{23}^{(3)};\qquad g_{33}^{(3)}=E_z^{(3)}. \qquad (9)$$

The potential energy of sandwich plates deformation is the sum of potential energies of deformation of the bearing layers and filler:

$$u = \frac{1}{2}a(\upsilon,\upsilon) = \frac{1}{2}\sum_{i=1}^{3}\int_{V(i)}\{\epsilon^{(i)}\}^T\{\sigma^{(i)}\}dV_i \qquad (10)$$

Here is

$$\{\sigma^{(n)}\} = \{\sigma_x^{(n)},\sigma_y^{(n)},\tau_{xy}^{(n)}\};\{\epsilon^{(n)}\} = \{\epsilon_x^{(n)},\epsilon_y^{(n)},\gamma_{xy}^{(n)}\},\ n = 1,2$$

$$\{\sigma^{(3)}\} = \{\tau_{xx}^{(3)},\tau_{yx}^{(3)},\sigma_z^{(3)}\};\{\epsilon^{(3)}\} = \{\gamma_{xz}^{(3)},\gamma_{yz}^{(3)},\epsilon_z^{(3)}\}.$$

Numerical solution method

We can approximate the deflection of n-th thin base layer within each subregion using an incomplete cubic polynomial [6]

$$\begin{aligned} w_{0h}^{(h)} &= w_1^{(n)}L_1 + w_2^{(n)}L_2 + w_3^{(n)}L_3 + a_1^{(n)}L_1^2L_2 + a_2^{(n)}L_1^2L_3 + \\ &+a_3^{(n)}L_2^2L_1 + a_4^{(n)}L_2^2L_3 + a_5^{(n)}L_3^2L_1 + a_6^{(n)}L_3^2L_2 + 2a_7^{(n)}L_1L_2L_3 \end{aligned} \qquad (11)$$

Where are

$$a_1^{(n)} = w_1^{(n)} - w_2^{(n)} - b_3\varphi_1^{(n)} - c_3\psi_1^{(n)}$$

$$a_2^{(n)} = w_1^{(n)} - w_3^{(n)} + b_2\varphi_1^{(n)} - c_2\psi_1^{(n)}$$

$$a_3^{(n)} = w_2^{(n)} - w_1^{(n)} + b_3\varphi_2^{(n)} + c_3\psi_2^{(n)}$$

$$a_4^{(n)} = w_{21}^{(n)} - w_3^{(n)} - b_1\varphi_2^{(n)} - c_1\psi_2^{(n)}$$

$$a_5^{(n)} = w_3^{(n)} - w_1^{(n)} - b_2\varphi_3^{(n)} - c_2\psi_3^{(n)}$$

$$a_6^{(n)} = w_3^{(n)} - w_2^{(n)} + b_1\varphi_3^{(n)} + c_1\psi_3^{(n)}$$

$$a_7^{(n)} = \frac{1}{4}\sum_{s=1}^{6}a_s^{(n)} \qquad (12)$$

Here L_i, i = 1, 2, 3 are the coordinates, given by [6]:

$$L_i = \frac{1}{2\Delta}(a_i + b_ix + c_iy) \qquad (13)$$

Where

$$a_1 = x_2y_3 - y_2x_3\ ;\quad b_1 = y_2 - y_3\ ;\quad c_1 = x_3 - x_2 \qquad (14)$$

The displacement of the median surfaces of bearing layers and the filler are presented in the form of linear polynomials :

$$u_{0h}^{(n)} = u_1^{(n)}L_1 + u_2^{(n)}L_2 + u_2^{(n)}L_2\ ; \qquad (15)$$

$$v_{0h}^{(n)} = v_1^{(n)}L_1 + v_2^{(n)}L_2 + v_2^{(n)}L_2\ ; \qquad (16)$$

$$w_h^{(3)} = w_1 L_1 + w_2 L_2 + w_3 L_3 \ .$$

Then the total potential energy of each triangle can be written in the local coordinate system in the form:

$$\hat{Э}(\hat{v}_h) = \sum_{n=1}^{3} \hat{U}_h^{(n)} - \sum_{n=1}^{2}\sum_{i=1}^{3}\left(F_{xi}^{(n)}\hat{u}_i^{(n)} + F_{yi}^{(n)}\hat{v}_i^{(n)} + F_{zi}^{(n)}\hat{w}_i^{(n)} + F_{\varphi i}^{(n)}\hat{\varphi}_i^{(n)} + F_{\psi i}^{(n)}\hat{\psi}_i^{(n)}\right) \qquad (17)$$

The total potential energy of the shell of the system will be the following:

$$Э(v_h) \equiv Э(\vec{v}) = \frac{1}{2}(K\vec{v}, v) - (\vec{f}, \vec{v}); \vec{v}, \vec{f} \in R \ , \qquad (18)$$

Where K is the positive definite symmetric matrix of order N and $\vec{f}$ is the N – dimension vector of nodal external load. Then the task of determining the stress-strain state structures can be represented as minimization of a quadratic functional

$$\vec{u} \in R^n; Э(\vec{u}) = \inf_{\vec{v} \in R^N} Э(\vec{v}) \qquad (19)$$

It is necessary to determine the natural frequencies and corresponding vibration modes during the calculating the stress-strain state of structural elements exposed to vibration. Finding the main natural frequency can be reduced to the minimization problem, where the functional is determined by the ratio of the Rayleigh-Ritz method:

$$\omega^2 = \min \qquad , v \in V. \qquad (20)$$

Here П(v) is the peak value of strain energy, T (v) – is a quantity which is proportional with the factor ω^2 to amplitude value of kinetic energy.

Using approximations (11), (15), (16) finite functional relations Rayleigh can be represented as

$$F(v_h) = \qquad ; \ v_h \in V. \qquad (21)$$

It is proposed to use the wise descent method (ICS) [7] for minimization of the functional (18), (21) in the present work. The choice of this method stems from the fact that its application is not necessary in the formation and storage of mass and stiffness matrices of large dimensions, the numbering of nodes for sampling of arbitrary, it greatly reduces the memory requirements of a PC. Wise descent method is iterative and sustainable method, where rounding errors have little influence on the accuracy of the final result. The к+1 approximation is constructed in the form

$$^{k+1} = \ ^{k} + \beta^{k+1}\lambda_i \ ^{k+1} \quad _{i};;$$

is a vector of unknown displacements; is the unit vector in the direction of the components $_i^k$; λ_i^{k+1} is the step; β is the parameter of acceleration of the setting of an iterative process [7].

Step size is found from the condition for maximum reduction of functionals (18), (21).

As a test the problem of bending and natural vibrations of layered panels of square, trapezoidal and rectangular shapes with different conditions of consolidation are solved. The results are compared with the results obtained on the basis of experimental, numerical and analytical methods [3, 8].Maximum number of triangular elements in the problems amounted was no more than 1000. Maximum number of iterations was less than 50. The error in the determination of deflection in the center of the plates was not more than 0,2%.Data on the identification of the main natural frequencies were compared with results obtained by an asymptotic method of Bolotin. After 120 iterations, the maximum error in determining the basic natural frequency was 4,3%.

Conclusions

In conclusion, we note the following positive aspects of proposed approach. In the present triangular element of the layered structure using various approximations of the displacement of bearing layers and filler allows you to simulate various types of fastening and joining of layered structures. Thus, one layer can be freely fixed, and the other is rigidly fixed, in one layer there can be bolted connection, but not in others, etc. In addition, through the use of these approximations movements, it was managed to reduce the order of the element in comparison with other finite element models [4, 5].

Due to the fact that when using the method of descent-wise is not necessary in the formation and storage of mass and stiffness matrices, the numbering of nodes for arbitrary sampling area, which significantly reduces the need for a computer memory. As the wise descent method is an iterative algorithm, rounding errors have a small effect on the accuracy of the final result.

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