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## **THE FREQUENCY ERROR OF THERMOELECTRIC VOLTAGE CONVERTERS**

### **Introduction**

To create the precision thermoelectric voltage converter in a range which is not supported by the state reference analytical and experimental methods are apply. For this implementation a model of schematic diagram of thermoelectric voltage converter is developing as a maximum approximate to real physical model [1, 2].

Frequency error is one of the most important in the thermoelectric reference. It is specified that reactive resistance component presences in voltage transducers and series resistances, connectives, commentators', etc.

For frequency error analysis the mathematical models and models in different electronic circuitry analysis packets are used.

The researcher chooses more useful analysis method for his own. But a model completeness guaranteeing is the most important.

For example, digital models make it possible to look through very quickly a large quantity of options.

The analytical models' advantage is an ability to complete analysis conduction; you can estimate influence of each coefficient, its value.

You can easily change model and implement the calculation if model is represented in electrical circuit form in Micro-Cap [3].

### **Problem definition**

As definition of physical quantity, reference has the best metrological performance, that's why there are no instruments or systems for reference certification. The certification is carried out theoretically – experimental method.

Basically the model created and the definitions of quantitative are underlying this method.

The thermoelectric voltage converter is the main module of alternating voltage reference. In the thermoelectric voltage converter several physical processes occurs each of them affects on metrological characteristics in AC voltage reproduction.

When creating a model of the precise thermoelectric voltage converter should be taken into account voltage conversion into the current, current in capability, capability in the quantity of heat, the quantity of heat into the temperature increment, temperature into electromotive force processes.

The frequency error is the most important error component in AC voltage reproduction.

By definition references have the top high metrological performance that's why more there are no precision instruments with the help of which you can determine these errors by experiment.

That's why the study objective is a creation of an analysis–experimental model which would make the adequate valuation of a frequency error possible for a wide range of an equipment with a high dispersion of measured values for elements of real circuits.

### Modeling in Micro–Cap

To estimate the frequency error was proposed this precision thermoelectric voltage converters model in Micro–Cap (fig. 1). This is the scheme with lumped RLC – parameters. In this model distributed n space parameters of inductance and capacitor, also active resistance components variation because of Foucault effect is not take into account due to the fact that frequency is viewed in a range of  $(30 \div 100)$ MHz.

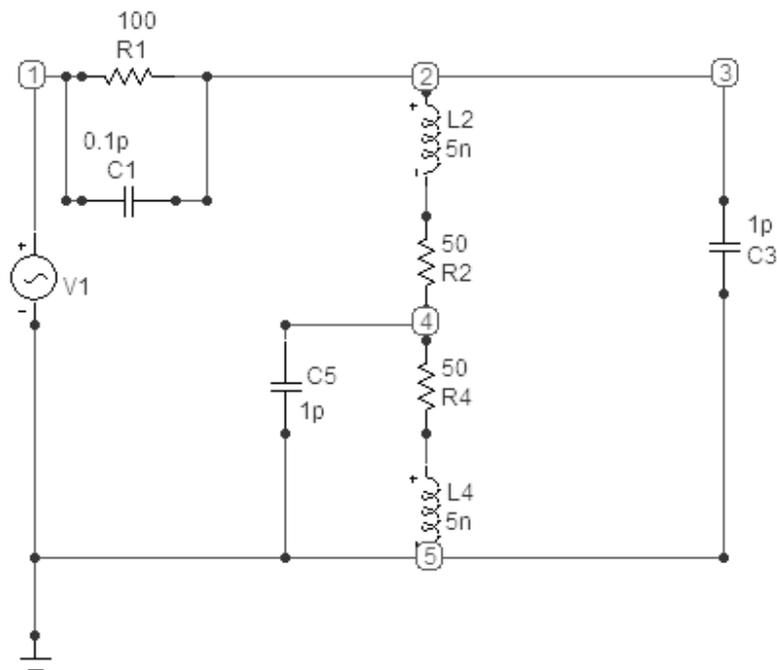


Fig. 1. Equivalent circuit of the precision thermoelectric voltage converter

The precision thermoelectric voltage converter is represented by  $R_1$ ,  $R_2$ ,  $R_3$  resistors links,  $C_3$  and  $C_5$  heater– thermo element which bridges a thermal converters heater and  $L_2$ ,  $L_4$  inductivities. Here:

$$R_2 + R_4 = R - \text{Heater resistance};$$

$$L_2 + L_4 = L - \text{Heater inductivity};$$

$C_3$  – the capacity bridging a heater;

$C_5$  – the capacity between heaters middle and thermal converter.

For next calculations these parameters values were measured on real thermal converters.

Modern thermal converters are divided into two big groups: one–elements and multielements.

The thermal converters isolation from heater on direct current is the constructions feature but between each pair of thermal converter and a heater is a capacity which is evaluated in (1÷3) pF. Through these capacities the current flows on a high frequency. It is the cause of the  $I_2, I_4$  currents are different in one–element thermal converters.

In multielement thermal converters a picture of currents is more difficult and it is necessary to discount all capacities and resistors values between each thermal converters capacities in equivalent scheme. The total efficiency, which is released on the heater, is equal to the sum of efficiencies on each heater-thermal converter links. One–element thermal converter includes two heater links through which  $I_2, I_4$  different currents will flow.

### **Application of analytical method for frequency error formula receipt**

Fractional error in the output unit in our case thermo electrical propulsive force (TEPF) can be expressed by next ratio:

$$\gamma(f) = \frac{E(f_i) - E(f_0)}{E(f_0)}, \quad (1)$$

there is  $E(f_i)$ – TEPF on a  $f_i$  frequency which is summary capability proportional allocated in the heater at a frequency  $f_i$

$$E(f_i) = k \left( [I_2(f_i)]^2 R_2 + [I_4(f_i)]^2 R_4 \right), \quad (2)$$

$E(f_0)$ – TEPF on a  $f_0$  frequency, calibration frequency is determined by the relative error or current, which creates a constant voltage.

Using thermal calibration  $f_0 = 0$

$$E(f_0) = k \left( [I_2(f_0)]^2 R_2 + [I_4(f_0)]^2 R_4 \right), \quad (3)$$

Based on (1), (2), (3) formulas obtain frequency error in expression:

$$\gamma(f) = \frac{E(f_i)}{E(f_0)} - 1 = \frac{[I_2(f_i)]^2 R_2 + [I_4(f_i)]^2 R_4}{[I_2(f_0)]^2 R_2 + [I_4(f_0)]^2 R_4} - 1. \quad (4)$$

In the above formulas  $I_2, I_4$  currents are the currents through the  $R_2, R_4$  resistances accordingly.

$I_2(f_i)$  – the root–mean–square current which flows through the active resistor  $R_2$ .

$I_4(f_i)$  – the root-mean-square current which flows through the active resistor  $R_4$ .

$I_2(f_0), I_4(f_0)$  – the currents on  $f_0$  frequency.

$I_2, I_4$  currents, which are included in the formula (4) can be found on the basis of Kirchoff's laws and Ohm's system of equations (5) for the equivalent circuit, shown in Fig. 2

$$\begin{cases} I_1 - I_2 - I_3 = 0, \\ I_2 - I_4 - I_5 = 0, \\ I_1 z_1 + I_2 z_2 + I_5 z_5 = u, \\ I_4 z_4 + I_5 z_5 = 0, \\ -I_2 z_2 + I_3 z_3 - I_4 z_4 = 0, \end{cases} \quad (5)$$

here  $z_1$  – the  $C_1$  capacity and the  $R_1$  resistance parallel connection,

$z_2, z_4$  – accordingly  $L_2, L_4$  inductivities and  $R_2, R_4$  resistances cascade connection.

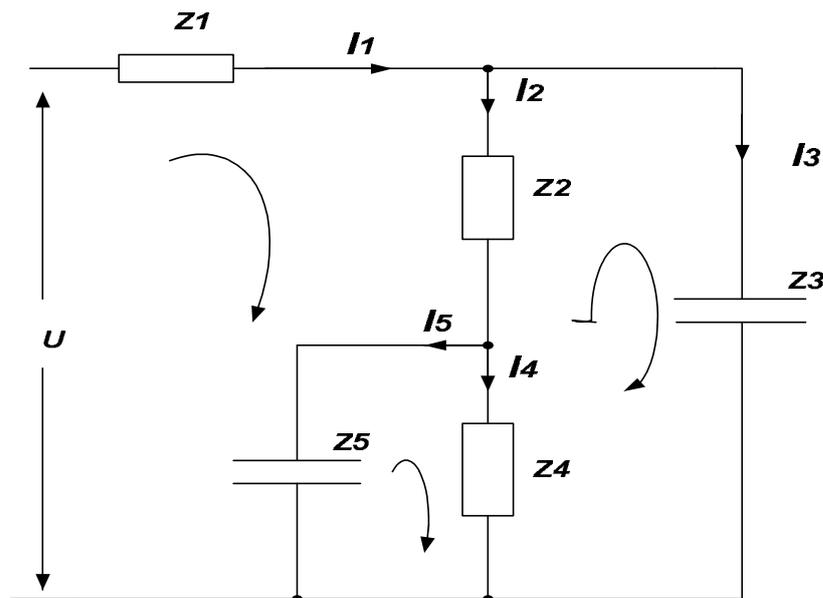


Fig. 2. Equivalent circuit of the thermal electric precision voltage converter

Given that:

$$z_1 = \frac{R_1 \cdot \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}, z_2 = R_2 + j\omega L_2, z_3 = \frac{1}{j\omega C_3}, z_4 = R_4 + j\omega L_4, z_5 = \frac{1}{j\omega C_5} \quad (6)$$

and these resistances are the time constants:

$$\tau_1 = R1 \cdot C1, \quad \tau_2 = \frac{L2}{R2}, \quad \tau_4 = \frac{L4}{R4}. \quad (7)$$

The matrix equation based on system of equations (5):

	1	2	3	4	5
1	1	-1	-1		
2		1		-1	-1
3	Z <sub>1</sub>	Z <sub>2</sub>			Z <sub>5</sub>
4				Z <sub>4</sub>	-Z <sub>5</sub>
5		-Z <sub>2</sub>	Z <sub>3</sub>	-Z <sub>4</sub>	

$$\times \begin{matrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{matrix} = \begin{matrix} 0 \\ 0 \\ U \\ 0 \\ 0 \end{matrix} \quad (8)$$

According to Cramer's rule  $I_2, I_4$  currents are finding as follow:

$$I_2(f) = \frac{\det 2}{\det c}, \quad (9)$$

that is:

$$[I_2(f)]^2 = \frac{\text{Re}^2(\det 2) + \text{Im}^2(\det 2)}{\text{Re}^2(\det c) + \text{Im}^2(\det c)}, \quad (10)$$

the same:

$$I_4(f) = \frac{\det 4}{\det c}, \quad (11)$$

that is :

$$[I_4(f)]^2 = \frac{\text{Re}^2(\det 4) + \text{Im}^2(\det 4)}{\text{Re}^2(\det c) + \text{Im}^2(\det c)}, \quad (12)$$

there is  $\det c$  – matrix system determinant, which equals next ratio:

$$\det c = \begin{vmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 \\ z_1 & z_2 & 0 & 0 & z_5 \\ 0 & 0 & 0 & z_4 & -z_5 \\ 0 & -z_2 & z_3 & -z_4 & 0 \end{vmatrix}, \quad (13)$$

$\det 2$  – matrix system determinant, in which the second column is replaced by the column-free members, it is:

$$\det 2 = \begin{vmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \\ z_1 & u & 0 & 0 & z_5 \\ 0 & 0 & 0 & z_4 & -z_5 \\ 0 & 0 & z_3 & -z_4 & 0 \end{vmatrix}, \quad (14)$$

det4 – matrix system determinant, in which the fourth column is replaced by the column-free members, it is:

$$\det 4 = \begin{vmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ z_1 & z_2 & 0 & u & z_5 \\ 0 & 0 & 0 & 0 & -z_5 \\ 0 & -z_2 & z_3 & 0 & 0 \end{vmatrix}, \quad (15)$$

Using the method of trajectories and applying algebra Grasmane [2], find:

$$\det c = -[(z_5 + z_4)(z_2 z_3 + z_1 z_3 + z_1 z_2) + z_4 z_5 (z_1 + z_3)], \quad (16)$$

$$\det 2 = -u z_3 (z_4 + z_5), \quad (17)$$

$$\det 4 = -U \cdot z_3 \cdot z_5. \quad (18)$$

Substitute the (16), (17), (18) expressions in formulas for finding  $I_2(f)$  and  $I_4(f)$ :

$$I_2(f) = \frac{\det 2}{\det c} = \frac{U \cdot z_3 (z_5 + z_4)}{(z_5 + z_4)(z_2(z_1 + z_3) + z_1 z_3) + z_4 z_5 (z_1 + z_3)}, \quad (19)$$

$$I_4(f) = \frac{\det 4}{\det c} = \frac{U \cdot z_3 z_5}{(z_5 + z_4)(z_2(z_1 + z_3) + z_1 z_3) + z_4 z_5 (z_1 + z_3)} \quad (20)$$

and find root-mean-square  $I_2(f_0)$ ,  $I_4(f_0)$  current value:

$$[I_2(f_0)]^2 = \left[ \frac{U}{R1 + R2 + R4} \right]^2, \quad (21)$$

$$[I_4(f_0)]^2 = \left[ \frac{U}{R1 + R2 + R4} \right]^2. \quad (22)$$

According to (4) formula and taking into account (6), (7), (10), (12), (19), (20), (21), (22) formulas we get an expression for finding the frequency error

consequently simplification in the MathCAD. For simplification of the formula note we divide it on some parts:

$$\gamma(f) = (1 + w^2 \tau_1^2)^2 \frac{(R2 + R4 + R1)^2}{R2 + R4} \times$$

$$\left[ \frac{\left[ (1 - w^2 \cdot C5 \cdot L4)^2 + (w \cdot C5 \cdot R4)^2 \right]}{\left[ (R2 + R4(1 + w^2 \tau_1^2) - w^2 \cdot R2 \cdot R4 \cdot C5(\tau_2 + \tau_4)(1 + w^2 \tau_1^2) + \right.} \right.$$

$$\left. \left. + R1 + w^2 \cdot C3 \cdot R1 \cdot R2(\tau_1 - \tau_2) + w^2 \cdot C3 \cdot R1 \cdot R2(\tau_1 - \tau_4) \right)^2 \right] +$$

$$\frac{\cdot R2 +}{\left[ w^2 \left[ (R2 \cdot \tau_2 + R4 \cdot \tau_4)(1 + w^2 \tau_1^2) + R2 \cdot R4 \cdot C5(1 - w^2 \cdot \tau_2 \tau_4)(1 + w^2 \tau_1^2) - \right. \right.} \right.$$

$$\left. \left. - R1 \cdot \tau_1 + C3 \cdot R1 \cdot R2(1 + w^2 \tau_1 \tau_2) - C3 \cdot C5 \cdot R1 \cdot R2 \cdot R4 \cdot w^2(\tau_2 - \tau_1 + \tau_4 + w^2 \tau_1 \cdot \tau_2 \cdot \tau_4) + \right. \right.$$

$$\left. \left. \frac{+ R4}{+ C5 \cdot R1 \cdot R4(1 + w^2 \tau_1 \tau_2) + C3 \cdot R1 \cdot R4(1 + w^2 \tau_1 \tau_4) \right]^2 \right] - 1.$$

Next diagrams show the whole range of the dependency of frequency error from frequency for several C1 capacity values and

$$R1 = 100\Omega, R2 = R4 = 50\Omega, \quad C2 = C5 = 1\text{ pF}, \quad L2 = L4 = 5\text{ nH}.$$

The C1 capacity values are selected in such a way that on 30 MHz frequency where frequency error spans in defined limits. Modeling is conducting in MathCad 14.

For analyses completeness we give a comparison table of frequency error value results which are derived by analytical way and in Micro-Cap interpreted results with C1=C3=C5=1 pF, since Micro-Cap has several advantages. But since in this package numerical differentiation and integration are used, which give less accurate results than the analytical method, the method will estimate the error by comparing.

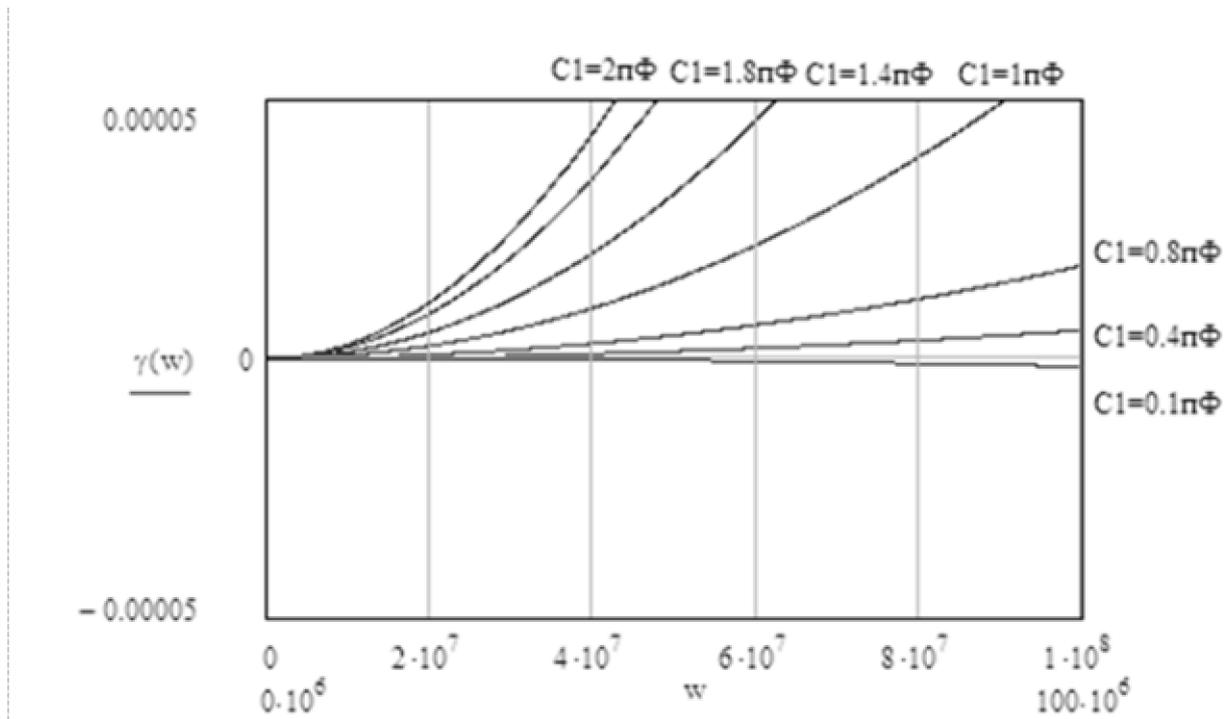


Fig. 3. Graph of frequency error from C1 capacity value dependence

**Table 1.**

Frequency error values when  $C1=C3=C5=1\text{pF}$

Frequency error	Frequency						
	$1 \times 10^{-13}$ , kHz	$10 \times 10^{-5}$ , MHz	$30 \times 10^{-4}$ , MHz	$100 \times 10^{-3}$ , MHz	$110 \times 10^{-3}$ , MHz	$200 \times 10^{-3}$ , MHz	300 MHz
MathCad modeling results	2.407	2.405	2.163	2.383	2.877	9.254	0.011
Micro-Cap 8 modeling results	1.787	1.78903	1.6115	1.8076	2.729	9.254	0.02
Methods error $\Delta$	0.62	0.61597	0.5515	0.5754	0.148	0	0.09

Micro-Cap 8 modeling result is shown on the frequency error depends on C1 changes graph:

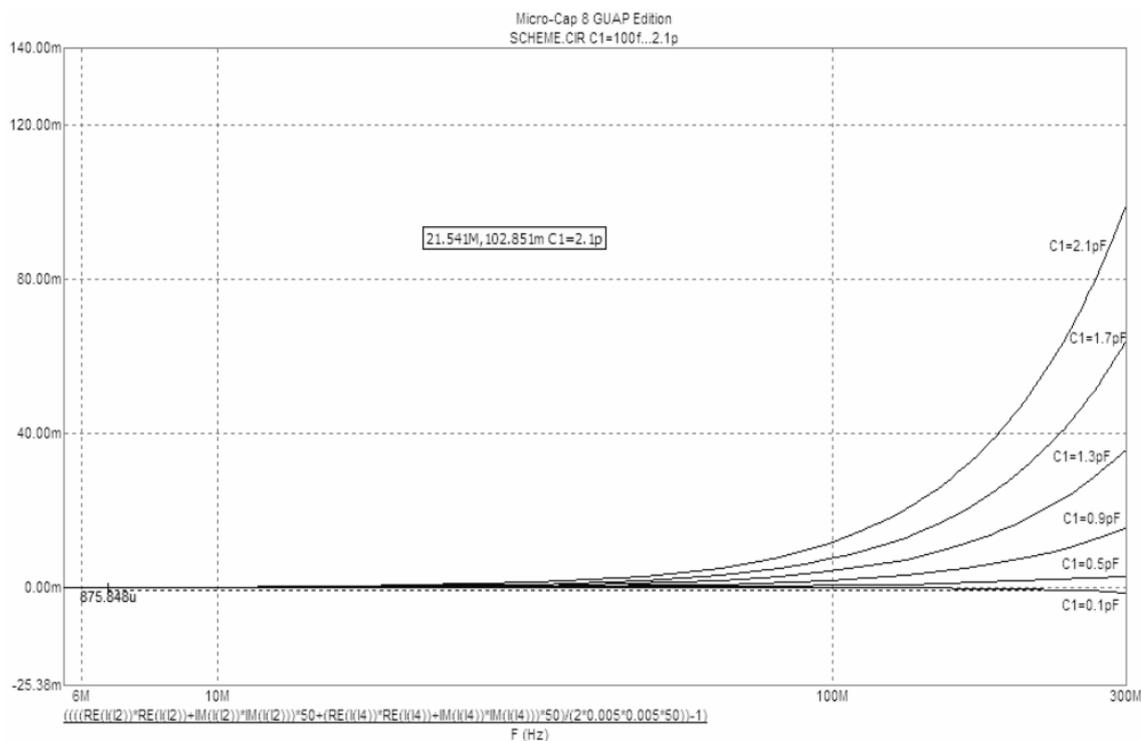


Fig. 4. Graphic presentation of Micro–Cap modeling result

## Conclusion

An expression finding analytical methods and the formula evaluation are giving exact results but an analytical method is more tedious. That's why we conclude that in the range till 100MHz for projection problems we can use Micro–Cap data.

For the automation and accumulation the process of analytical method frequency errorfinding an increasing was created the program in MatLab 7.7.0 which gives identical results obtained in other packages.

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