UDK 539.3.

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APPLICATION OF 3-LAYER FINITE ELEMENT ANALYSIS OF STRESS-DEFORMED STATE OF THE STRUCTURE

INTRODUCTION

Multilayer structures are widespread in technology. In some cases, using of multilayer structures is dictated by the desire to combine lightness with sufficient strength and rigidity. There are three-layered plates and shells with soft filler as example. The normal stresses in bending are seen mostly in extreme (carrying) layers; filler acts as a bond between these layers and works primarily in shear. In other cases, a multilayer structure used in connection with the need to combine various protective properties. As an example we mention the sandwich wall panels of civilian buildings, which combine mechanical strength, thermal insulation and sound quality. The sandwich plates and shells have a special place among the multi-load bearing structures. [1, 3] .

Application of high-strength steels, titanium and its alloys, reinforced plastics and other composite materials based on heavy-duty continuous fibers or whiskers in a thin-walled reinforced constructions, working in conditions of bending and compression, it is often ineffective. This is because of the condition of structural strength of these materials should have a very small thickness. But it is sharply reduced moment of inertia of the plate or shell, and the design, especially at low modules of elasticity of the material has a low critical voltage instability.

This deficiency deprived of sandwich plates and shells. A three-layer plate or shell consists of two relatively thin outer layers of high strength materials, coupled fiber filler, whose thickness is much greater than the thickness of the bearing layers. Strength properties and density of the filler, usually much lower than carrier layers.

Problem Statement

Stresses and strains in the triangular element

Divide the plate into triangular elements and define the coordinate system as follows (Fig. 1):

$X; Y; Z$ – global Cartesian coordinate system;

X', Y', Z' – local orthogonal coordinates of the element;

$\xi; \eta; \varsigma$ – L – coordinates of each element.

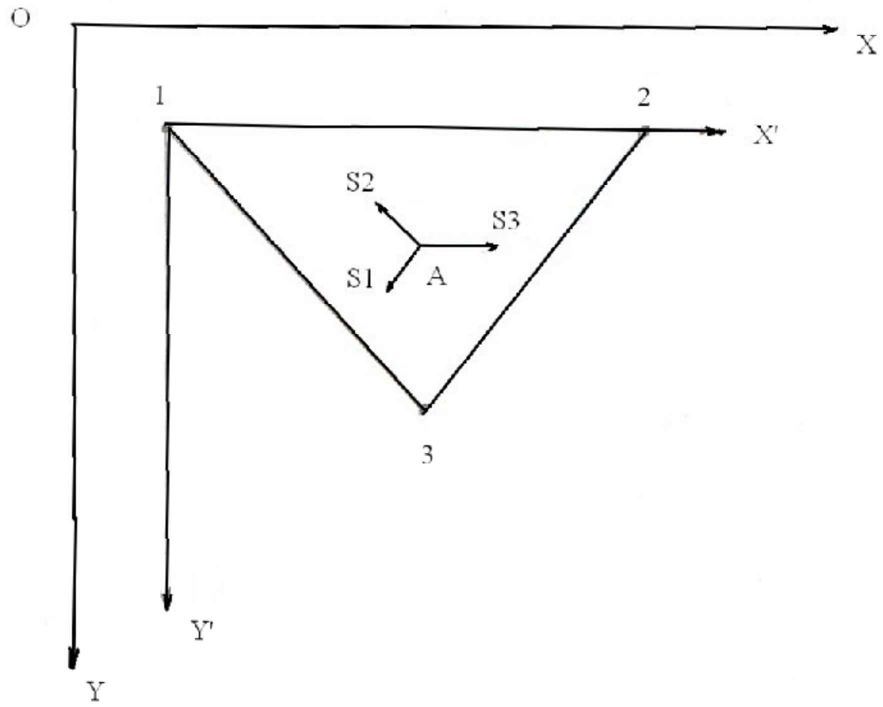

Fig. 1. The system of coordinate axis

There exist the following relationship between ($x; y$) and (ξ; η; ς)

$$\begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ x_1 & x_1 & x_1 \\ y_1 & y_1 & y_1 \end{pmatrix} \begin{pmatrix} \xi \\ \eta \\ \varsigma \end{pmatrix} \tag{1}$$

Denoting the displacement in the plane of the directions $S_{1\text{-}3}$, u_{1-3}, we obtain

$$u_1 l_1 + u_2 l_2 + u_3 l_3 = 0, \tag{2}$$

where l_{1-3} – the lengths of the finite element.

Deformation can be expressed as

$$\boldsymbol{\varepsilon}_1 = (\varepsilon_1; \varepsilon_2; \varepsilon_3) = \left\{ \frac{\partial u_1}{\partial s_1}; \frac{\partial u_2}{\partial s_2}; \frac{\partial u_3}{\partial s_3} \right\} \tag{3}$$

$$\boldsymbol{\varepsilon}_x = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \alpha(\beta+\gamma) & \beta(\alpha+\gamma) & -\alpha\beta \\ -(\beta+\gamma) & (\alpha+\gamma) & \beta-\alpha \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} \tag{4}$$

Abbreviated $\boldsymbol{\varepsilon}_x = A\boldsymbol{\varepsilon}_1$; α, β, γ cotangent vertical angles related by

$$\alpha\beta + \beta\gamma + \gamma\alpha = 1.$$

The components of the stress $\sigma_x = \{\sigma_x, \sigma_y, \tau_z\}$ contact the strains $\boldsymbol{\varepsilon}_x = \{\varepsilon_x; \varepsilon_y; \varepsilon_z\}$ by Gook's law: $\boldsymbol{\sigma}_x = \boldsymbol{H}_x \boldsymbol{\varepsilon}_x$, where

$$H_x = \frac{E}{1-\upsilon^2}\begin{pmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \mu \end{pmatrix} \tag{5}$$

for an isotropic material, E is the shear modulus, υ is the Poisson's ratio.

Shear strain

$$\boldsymbol{\gamma}_x = \frac{1}{2S}\begin{pmatrix} y_1 & y_2 & y_3 \\ -x_1 & -x_2 & -x_3 \end{pmatrix}\begin{pmatrix} l_1 & 0 & 0 \\ 0 & l_2 & 0 \\ 0 & 0 & l_3 \end{pmatrix}\boldsymbol{\gamma}_1 , \tag{6}$$

where $\boldsymbol{\gamma}_1 = \{\gamma_1, \gamma_2, \gamma_3\}$ and $\gamma_1 l_1 + \gamma_2 l_2 + \gamma_3 l_3 = 0$.

For components $\tau_x = \{\tau_x; \tau_y\}$

$$\boldsymbol{\tau}_x = \boldsymbol{G}_x \boldsymbol{\gamma}_x ; \quad \boldsymbol{G}_x = \begin{pmatrix} G & 0 \\ 0 & G \end{pmatrix}. \tag{7}$$

Similarly, describe the voltage $\boldsymbol{\tau}_{1e}$

$$\boldsymbol{\tau}_{1e} = \boldsymbol{G}_1 \boldsymbol{\gamma}_{1e};$$

where

$$\boldsymbol{G}_1 = \frac{G}{2S}\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}; \qquad \boldsymbol{\gamma}_{1e} = \begin{pmatrix} l_1 & 0 & 0 \\ 0 & l_2 & 0 \\ 0 & 0 & l_3 \end{pmatrix}\boldsymbol{\gamma}_1. \tag{8}$$

Function of displacement and the stiffness matrix

For the flexural deformation of the element is assumed that the displacement in the plane are proportional to the distance from the middle surface

$$u_{1-3} = z\phi_{1-3}. \tag{9}$$

The rotations in element reflects on figure 2:

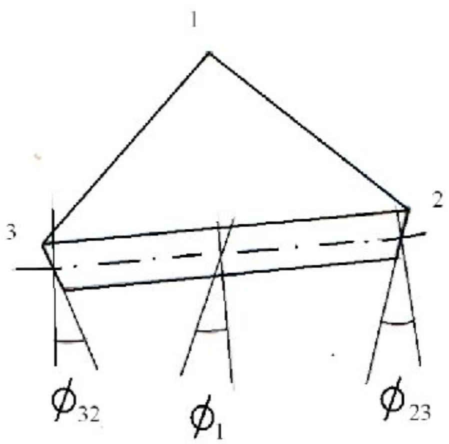

Fig. 2. Rotation in finite element

According to equation (2) we have

$$\phi_1 l_1 + \phi_2 l_2 + \phi_3 l_3 = 0.$$

It is assumed that ϕ ($\phi = \{\phi_1;\ \phi_2;\ \phi_3\}$) is expressed by a polynomial 1-st order [4] .

Then

$$w = w_1\xi + w_2\eta + w_3\zeta + c_1\eta\zeta + c_2\zeta\xi + c_3\xi\eta, \tag{10}$$

where w_{1-3} – deflection peaks 1-3.

To determine c_{1-3} suggest that the strain components $\gamma_i l_i$ are constant on each side;

$$c_1 = -(\phi_{23} + \phi_{32})\frac{l}{2}. \tag{11}$$

Calculated similarly c_2 and c_3.

Thus, the deformation element can be described by nine nodal displacements

$$u_e = \{w_1; w_2; w_3;\ \phi_{12} l_3; \phi_{13} l_2; \phi_{23}\ l_1; \phi_{21} l_3; \phi_{31} l_2; \phi_{32} l_1.\} \tag{12}$$

Inner really work through the actual movement of the element is expressed as follows:

$$\overline{W}_B = \int (\tilde{\varepsilon}_1 H_1 \varepsilon_1 + \tilde{\gamma}_1 G_1 \gamma_{1e}) dV = \tilde{\Phi}_e k_f \Phi_e + \tilde{w} k_c w. \tag{13}$$

Symbol $\sim$ means the transposed matrix,

$$\Phi_e = \{(\phi_{23} + \phi_{32})l_1; (\phi_{31} + \phi_{13})l_2;\ (\phi_{12} + \phi_{21})l_3\};$$

$$w = \{w_3 - w_2; w_1 - w_2; w_3 - w_1;$$

$$(\phi_{23} - \phi_{32})l_1;\ (\phi_{31} - \phi_{13})l_2; (\phi_{12} - \phi_{21})l_1\}.$$

The equations can be analyzed plane bending of laminated plate.

Conclusions

The obtained shape functions and the proposed approximation of displacements in the triangular finite element make it possible to accurately describe the stress-strain state in a triangular element with shear deformation in the soft layer and rotational inertia. Resulting element is simple and allows us to solve the problem for layered structures with a complete view of boundary conditions, which are defined in terms of displacements.

Test problems have shown a rather high convergence of solutions.

References

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