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V. A. Apostolyuk

OPTIMAL SENSOR NOISE FILTERING FOR CORIOLIS VIBRATORY GYROSCOPES

Introduction

Many modern angular rate sensors operate using sensing of the Coriolis force induced motion in vibrating structures. Such approach allows to avoid using expensive means of mechanisation as well as to increase long term reliability of sensors. Another benefit lays in the possibility to fabricate sensitive elements of such gyroscopes in miniature form by using modern microelectronic mass–production technologies. Such gyroscopes are frequently referred to as MEMS (Micro–Electro–Mechanical–Systems) gyroscopes. Being based on sensing of Coriolis acceleration due to the rotation in oscillating structures, Coriolis vibratory gyroscopes (CVGs) have a lot more complicated mathematical models, comparing to the conventional types of gyroscopes. One of such complication is a result of the useful signal proportional to the external angular rate being modulated with the intentionally excited primary oscillations [1–3]. From the control systems point of view, conventional representation of CVGs incorporates primary oscillation excitation signal as an input to the dynamic system, and unknown angular rate as a coefficient of its transfer functions [3]. As a result, conventional control and filtering systems design is practically impossible. At the same time, performances of CVGs are limited mainly due to the low signal–to–noise ratios. In view of this problem, optimal noise filter development is highly necessary.

This paper demonstrates synthesis of an optimal sensor noise filter using Wiener approach. Contrary to the dynamic Kalman filtering approach, Wiener filters allow to be implemented using simple analogue electronics and yet be as efficient for the stationary sensor noises.

Problem formulation

In order to be able to synthesise optimal filters for CVGs the following major steps must be completed: *a*) development of the mathematical model in demodulated signals, *b*) obtaining system transfer functions where angular rate is an input, *c*) analysis of stochastic disturbances affecting performances of CVGs, *d*) synthesis of optimal filters based on the obtained earlier transfer functions with respect to the spectral characteristics of stochastic disturbances, and finally, *e*) numerical simulations proving the performances of the optimal filters.

Demodulated dynamics and transfer function of CVGs

In the most generalized form, motion equations of the CVG sensitive element both with translational and rotational motion could be represented in the following form [4]:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 + g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 - g_2\Omega\dot{x}_1 - \dot{\Omega}x_1 = q_2(t). \end{cases} \quad (1)$$

Here x_1 and x_2 are the generalized coordinates that describe primary (excited) and secondary (sensed) motions of the sensitive element respectively, k_1 and k_2 are the corresponding natural frequencies, ζ_1 and ζ_2 are the dimensionless relative damping coefficients, Ω is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, q_1 and q_2 are the generalized accelerations due to the external forces acting on the sensitive element. The remaining dimensionless coefficients are different for the sensitive elements exploiting either translational or rotational motion. For the translational sensitive element they are $d_1 = d_2 = 1$, $d_3 = m_2/(m_1 + m_2)$, $g_1 = 2m_2/(m_1 + m_2)$, $g_2 = 2$, where were m_1 and m_2 are the masses of the outer frame and the internal massive element. In case of the rotational motion of the sensitive element, these coefficients are the functions of different moments of inertia (for greater details see [4]).

In the presented above motion equations, the angular rate is included as an unknown and variable coefficient rather than an input to the double oscillator system. Conventional control systems representation of such a dynamic system is shown in Fig. 1.

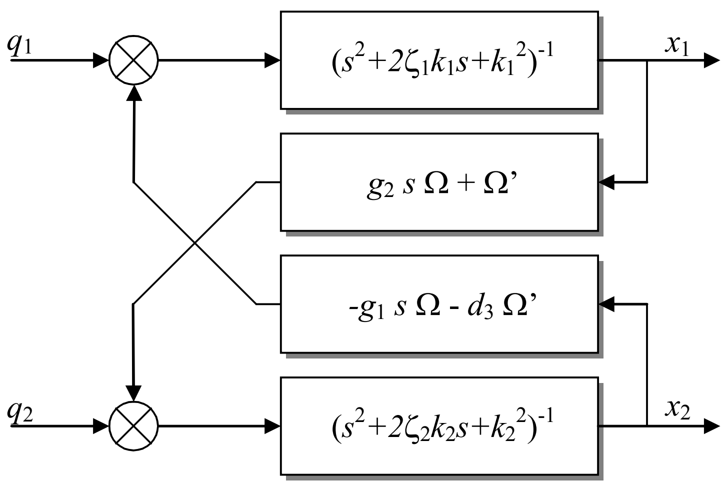

Fig. 1. Conventional representation of CVGs in control systems

In order to make the equations (1) suitable for to the transfer function synthesis one must make the following assumptions: angular rate is small comparing to the primary and secondary natural frequencies so that

$$k_1^2 >> d_1\Omega^2, \quad k_2^2 >> d_2\Omega^2 \tag{2}$$

and rotational and Coriolis accelerations acting along primary oscillation axis are negligible in comparison to the accelerations from driving forces

$$g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 << q_1(t). \tag{3}$$

With the assumptions (2) and (3) system (1) can be accurately represented with following transfer function for the secondary oscillations amplitude with respect to the input angular rate [5–7]:

$$W_{20}(s) = \frac{A_{20}(s)}{\Omega(s)} = \frac{q_{10}g_2}{4k^2\zeta(s+k\zeta)}. \tag{4}$$

Here $A_{20}(s)$ is the Laplace transformation of the secondary oscillations amplitude, $\Omega(s)$ is the input angular rate, q_{10} is the amplitude of the primary oscillations excitation accelerations. The following additional assumption were made, such as equal primary and secondary natural frequencies ($k_1 = k_2 = k$), equal damping ratios ($\zeta_1 = \zeta_2 = \zeta$), resonance excitation ($\omega = k$), and constant angular rate.

Transfer function (4) relates angular rate to the secondary oscillations amplitude. However, more appropriate would be to consider transfer function relating unknown input angular rate to the measured angular rate, which can be easily obtained from (4) by dividing it on the steady state scale factor. The resulting transfer function is

$$W(s) = \frac{k\zeta}{s+k\zeta}. \tag{5}$$

Although this case appears to be very specific, it still approximates transient process of a "tuned" CVG with accuracy suitable for most of applications [5, 7].

Stochastic sensor noise

Performances of CVGs can be affected by uncontrolled stochastic influences in two ways: as a "sensor noise", which is added to the output of the system, and as a "process noise" or disturbances, which are added to the input of the system. The former case is shown in the figure 1.

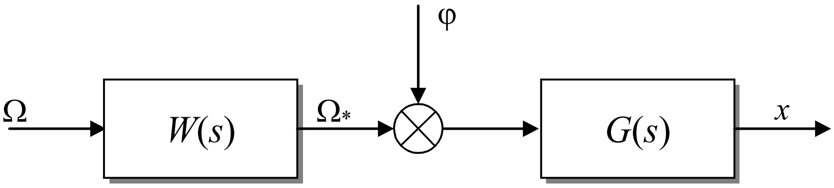

Fig. 2. CVG with added sensor noise and optimal filter

Here $W(s)$ is the system transfer function given by (5), φ is the stochastic sensor noise added to the CVG output, Ω is the angular rate, $G(s)$ is the optimal filter yet to be developed, x is the filtered output of the system, which in ideal case is equal to the angular rate Ω.

Assuming that CVG is installed on a moveable object, such as aircraft or land vehicle, its power spectral density can be represented as

$$S_{\Omega}(s) = \frac{\sigma^2 B^2}{B^2 - s^2}, \tag{6}$$

where B is the moveable object bandwidth. In this case sensor noise can be represented by the white noise as follows

$$S_{\phi}(s) = \gamma^2 \sigma^2 . \tag{7}$$

Here γ is the noise to angular rate ratio ("noise–to–signal" ratio). While using white noise as a model of sensor noise is quite common, in some cases model of the noise must be more sophisticated, that is to represent high pass noise present above the bandwidth of the moveable object, as follows:

$$S_{\phi}(s) = -\frac{\gamma^2 \sigma^2 s^2}{B^2 - s^2}. \tag{8}$$

Power spectral densities (7) and (8) cover most of the present in CVG cases of stochastic sensor noises. Nevertheless, other specific spectral densities can be taken into account and used in the presented below optimal filter synthesis procedure.

Optimal filter synthesis algorithm

General algorithm of the optimal filter synthesis for the system in Fig. 2 has been demonstrated in [8], with respect to the stationary stochastic sensor noise.

Error of the system is defined as a difference between the actual output of the system x and the ideal output, which is the given by the desired transformation $H(s)$ of the input:

$$\varepsilon = x - H(s) \cdot \Omega .$$

It is also assumed that signals x and Ω are the centred stochastic processes with known spectral densities $S_{\Omega}(s)$, $S_{\phi}(s)$, $S_{\Omega\phi}(s)$, and $S_{\phi\Omega}(s)$.

Performance criterion for the system is assumed to be in the form of the following functional:

$$J = E\{\varepsilon' \cdot R \cdot \varepsilon\} = \frac{1}{j} \int_{-j\infty}^{j\infty} tr(S'_{\varepsilon\varepsilon} \cdot R) ds . \qquad (9)$$

Here R is the weight matrix, and $S'_{\varepsilon\varepsilon}(s)$ is the transposed matrix of the error spectral densities. Using Wiener–Khinchin theorem we can calculate the error spectral density from the system transfer functions and signal spectral densities as follows:

$$\begin{aligned} S'_{\varepsilon\varepsilon}(s) = {} & (GW - H) S'_{\Omega} (W_* G_* - H_*) + (GW - H) S'_{\phi\Omega} G_* \\ & + G S'_{\Omega\phi} (W_* G_* - H_*) + G S'_{\phi} G_*, \end{aligned} \qquad (10)$$

where "*" designates Hermite conjugate. By means of introducing new variables defined as

$$\begin{gathered} DD_* = W S'_{\Omega} W_* + W S'_{\phi\Omega} + S'_{\Omega\phi} W_* + S'_{\phi} , \\ \Gamma\Gamma_* = R , \;\; G_0 = \Gamma G D , \\ T = \Gamma H (S'_{\Omega} W_* + S'_{\phi\Omega}) D_*^{-1} , \end{gathered} \qquad (11)$$

and substituting power spectral density (10) into (9), first variation of the performance criterion (9) with respect to the unknown filter related function G_0 will be

$$\delta J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr[(G_0 - T)\delta G_0 + \delta G_{0*}(G_{0*} - T_*)] ds . \qquad (12)$$

Minimum of the performance criterion is achieved when first variation (12) is zero. Apparently, this is achieved when [8]

$$G = \Gamma^{-1} (T_0 + T_+) D^{-1} . \qquad (13)$$

Here T_0 is the integral part of the matrix T, and T_+ is the part of the matrix T that contains only poles with negative imaginary part. These matrices are the result of the Wiener separation procedure.

Spectral densities (7) and (8) along with the suggested angular rate spectral density (6) can now be used to derive optimal filters based on the formula (13). After performing transformations according to (11), the optimal filters are found as:

$$G(s)=\frac{B\sqrt{1+\gamma^2}(s+\zeta k)}{\gamma s^2+s\sqrt{\gamma(B^2\gamma+\zeta^2k^2\gamma+2\zeta kB\sqrt{1+\gamma^2})}+\zeta kB\sqrt{1+\gamma^2}}, \qquad (14)$$

in case of the "white–noise" output added sensor noise, and

$$G(s)=\frac{B(s+\zeta k)}{s^2\gamma+s\sqrt{\zeta k\gamma(2B+\zeta k\gamma)}+B\zeta k}, \qquad (15)$$

in case of the "high–pass" sensor noise. Depending on which of the noise model is found to be the most appropriate, either filter (14) or filter (15) should be used.

Numerical simulations

Let us now study performances of the obtained optimal filters (14) and (15) in numerical simulations of the realistic CVG. In order to obtain the most realistic simulation results, equations (1) were used to build a numerical model of CVG dynamics using Simulink/Matlab. Resulting sensitive element model is shown in the figure 3.

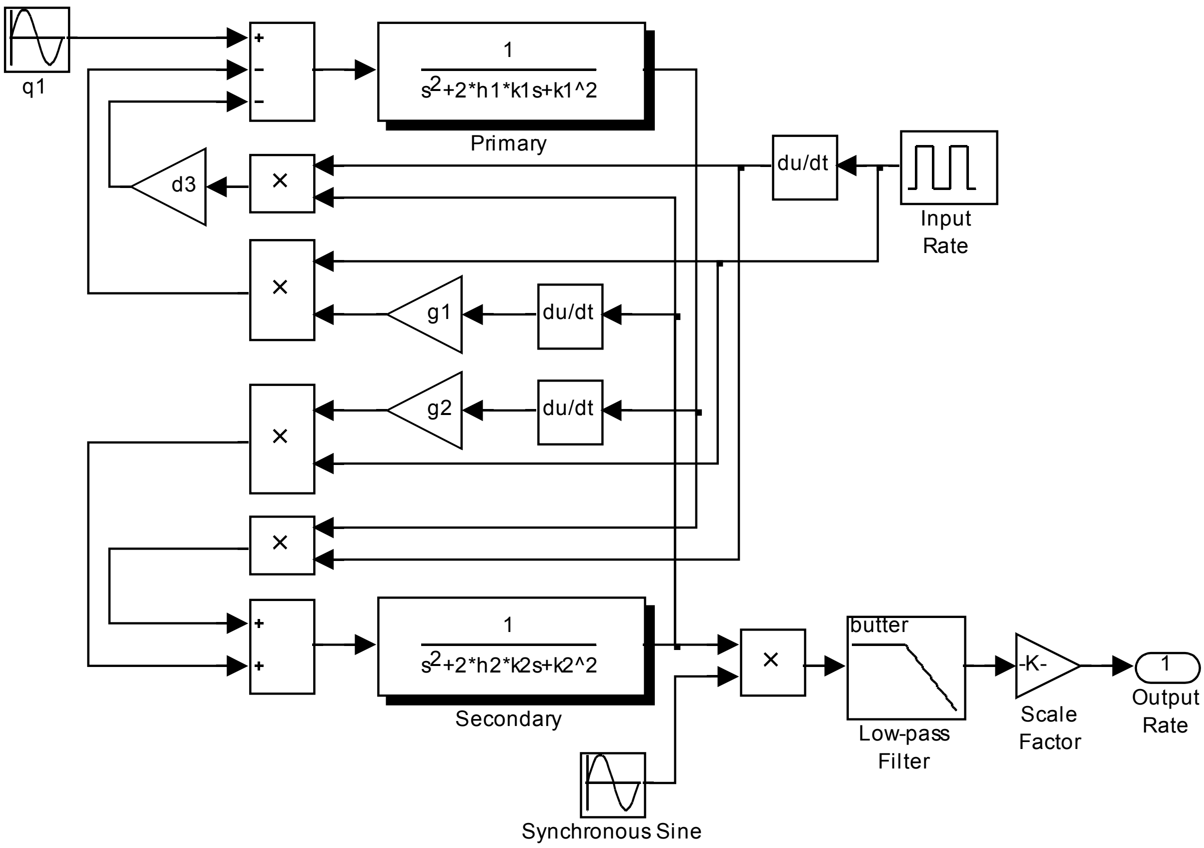

Fig. 3. Realistic CVG simulation model

In this model centrifugical accelerations were neglected and synchronous demodulator is added. Input angular rate is assumed to be in a form of square pulses. Results of numerical simulations of the “white” sensor noise filtering are shown in the figure 4.

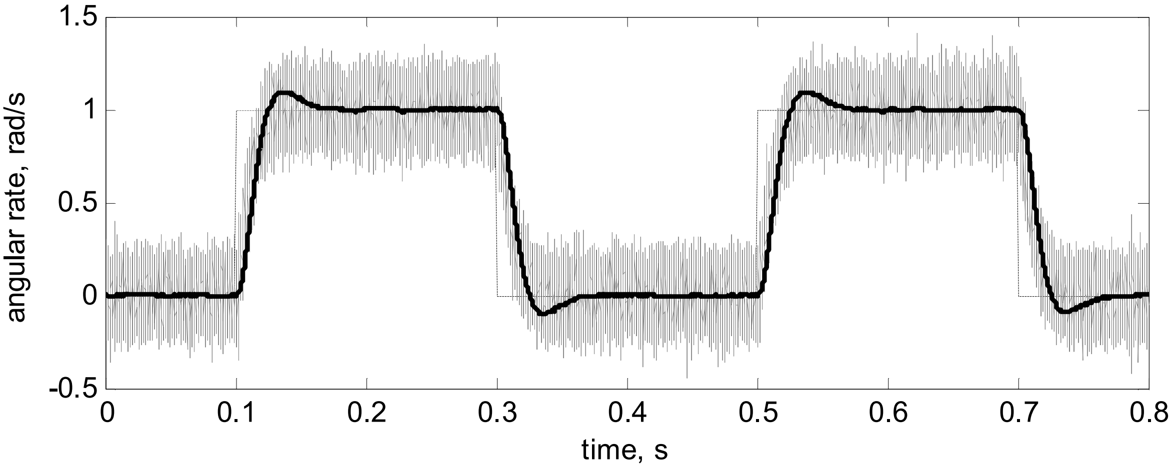

Fig. 4. Sensor noise filtering simulations
(dashed – input angular rate, gray – noised output, black – filtered output)

These simulations are performed for the γ=0.1 and bandwidth of the angular rate B=3 Hz. One should observe excellent performance of the synthesised filters.

Conclusions

Presented above synthesis of the filters of stochastic sensor noises resulted in two static filters capable of improving the performances of Coriolis vibratory gyroscopes in case of “white” and “high–pass” sensor noise. The latter has been demonstrated using explicit numerical simulations. The further analysis of the sensitivity of the filters performances in case of varying parameters of gyroscopes is viewed as a possible future development of the current research.

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